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ON THE DESIGN OF TOTAL WATER USE-BASED
INCENTIVE SCHEMES FOR SUSTAINABLE
GROUNDWATER MANAGEMENT

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Abstract

This paper sheds light on the design of various incentive schemes to face the unsustainable groundwater over-pumping by farmers. The response of the Water Authority in tackling this over-exploitation will differ according to whether it uses an incentive scheme based on the individual farmer's water use, which is his/her own private information, or it resorts to a total-water-use-based incentive schemes, where the total water use is publicly observable. Two schemes will be discussed. The first one corresponds to the framework of moral hazard in team problems where the Water Authority administers incentive schemes that do not balance the budget, thereby restoring water use efficiency. In the second scheme, the WA promotes the cooperative behavior. We show how cooperative management institutions can reduce water overuse and improve incentives for efficient water use, by inducing peer monitoring by cooperative members. We show that water overuse is more likely when punishments are weak and cooperatives are large. We also extend the basic analysis to allow for collusion in monitoring between cooperative members and compare different monitoring structures.

ملخص

تلقي هذه الورقة الضوء على عملية وضع مجموعة من الخطط التحفيزية المختلفة لمواجهة الإفراط في ضخ المياه الجوفية غير المستدامة من قبل المزارعين. وما من شك أن الأسلوب الذي تتبعه السلطات المسؤولة عن المياه في معالجة هذا الاستغلال المفرط سوف ينقسم إلى طريقتين: الطريقة الأولى أن تستخدم خطة تحفيزية قائمة على استخدام المياه عن طريق المزارع الفرد الذي يملك هو المعلومات الخاصة بكميات المياه التي يستهلكها، الطريقة الثانية أن تلجأ هذه السلطات إلى مجموعة من الخطط التحفيزية القائمة على الاستخدام الكلي للمياه، حيث انه يمكن مراقبة الاستخدام العام للمياه. وسوف يتم عرض خطتين للمناقشة. الأولى تتوافق مع إطار المخاطر المعنوية في حصر المشاكل المتشابهة، والتي تحاول سلطات المياه من خلالها أن تحقق التوازن في الخطط التحفيزية التي لا تحقق التوازن في الميزانية، مما يؤدي إلى استعادة كفاءة استخدام المياه. أما في الخطة الثانية، فتقوم سلطات المياه بتعزيز وتشجيع السلوك التعاوني. سوف نوضح كيف أن المؤسسات القائمة على الإدارة التعاونية يمكن أن تقلل من الاستخدام المفرط للمياه وتقوم بتحسين الحوافز التي تؤدي إلى الاستخدام الأمثل للمياه وذلك عن طريق تحفيز الدور الرقابي لدى جميع أعضاء الجمعيات التعاونية. كما نبين أيضا أن احتمالات الإفراط في استخدام المياه تصبح أكثر حدوثا عندما تكون العقوبات ضعيفة وتكون المناطق التي تديرها الجمعيات التعاونية كبيرة المساحة. ونقوم في هذه الورقة بتوسيع التحليل الأساسي وذلك للسماح للتعاون فيما بين أعضاء الجمعيات التعاونية في القيام بالدور الرقابي وكذلك المقارنة بين مختلف نظم الرقابة.

1. Introduction

Groundwater is a very important resource for at least two reasons: Firstly, it is appropriate for drinking water due to its generally high quality. Secondly, groundwater reservoirs constitute very important long-term storages that are particularly useful in arid zones like the MENA region.

Despite all the research efforts worldwide, unfortunately we cannot artificially recharge the groundwater on a large scale. This is the main reason why we can only raise the abstraction to a maximum of the long-term natural recharge. It is possible to over-pump any aquifer temporarily during very dry periods but overexploitation leads to irreversible degradations of this vital resource, due to saline intrusions and quality deteriorations generally generated by the water table decreasing.

A sustainable management of groundwater is then necessary, especially in dry regions where it often constitutes the only resource to maintain human life. Knowing that total discharge must be necessarily lower than the recharge (the renewable quantity); hence, sooner or later, the abstraction must fit to the water availability. So, the common sense tells us that it is wise to respect this constraint freely by moving smoothly to the target and not wait until nature imposes her law with all the drawbacks of a brutal adaptation.

As Negri said, in his famous 1989 paper, a collective decision in the common pool problems, like the groundwater management, leads to a better solution than the sum of all the independent individual decisions. So all the research in this important area shows that groundwater is a resource which rewards partnerships and punishes egoism. Our research will be focused on this idea and our main objective will be to formalize this problem on the basis of modern microeconomics.

This paper sheds light on the design of various incentive schemes to face the unsustainable groundwater over-pumping by farmers. The response of the Water Authority (WA) in tackling this over-exploitation will differ according to whether it uses an incentive scheme based on the individual farmer's water use which is his/her private information (namely the centralized water management), or it resorts to total-water-use-based incentive schemes, where the total amount of water used by farmers is publicly observable. For the latter, two sub-schemes will be proposed. The first one corresponds to the framework of moral hazard in team problems, where the payment for water by the team members is based on the whole team's water use, rather than on each individual's water use. In the second scheme, the WA makes use of the informational advantages farmers have over the WA because of their long standing and high trade links (especially in close-knit societies), namely through the implementation of cooperatives characterized by a collective responsibility rule, thus making all farmers jointly responsible for aggregate quantities of water used.

In the first total-water-use-based incentive scheme the WA administers incentive schemes that do not balance the budget, restoring thereby water use efficiency (Holmstrom, 1982). Such a scheme works independently of the team size, but it may be infeasible when farmers have endowment constraints. This is why one may resort to an alternative team or group incentive scheme that would not violate individual endowment constraints, namely through the implementation of cooperatives characterized by joint responsibility for water use. We show that this feature is likely to induce peer monitoring by cooperative members¹, which is likely a more efficient mechanism to reducing the overuse of water than the central monitoring applied by more centralized management structures. We in particular show that the overuse of water is more likely when monitoring costs are high and punishment levels are

¹ There is now a substantial literature on peer monitoring. See Stiglitz (1990), Besley and Coate (1995), Armanderiz de Aghion (1999), Ghatak & Guinnane (1999), Che (2002), Conning (2005).

weak. Moreover, straightforward comparison between the two total water use-based incentive schemes shows that with a sufficiently stringent punishment rate, cooperatives use less water than the efficient level, meaning that cooperatives preserve the resource more than in the full information setting. The intuition is that in this problem the preservation of the resource should be the primary objective of any policy in the short run since the exhaustion of the resource constitutes a real threat when the rate of utilization exceeds the rate of replenishment. This objective seems to conflict with social welfare considerations in the short run, however this is not the case in the long-run perspective, since the two objectives coincide.

The results in the cooperative setting are obtained for given levels of punishment and cooperative size, but cooperatives are typically able to influence both of these variables, and will do so in response to conditions that create a risk of the resource overuse—depending on how large the costs of monitoring are. We address the issue of optimal cooperative size using a numerical example and show that cooperatives can neither be too small because of "monitoring cost savings" effect nor too large because of "water overuse" effect. We extend our analysis thereafter to tackle the issue of collusion in monitoring efforts and demonstrate that the collusive monitoring effort is efficient because of the purely distributional character of peer monitoring. Third, we compare among different monitoring structures, mutual and rotating monitoring. Although in practice the mutual monitoring structure—whereby each farmer in the cooperative is being simultaneously monitored by all of her peers—is commonly observed, other monitoring structures deserve consideration. An interesting departure from the mutual structure is the "rotating monitoring" structure in which every farmer monitors only one of her peers, say her left neighbor. We show that the rotating monitoring effort equilibrium is higher than twice the mutual monitoring effort equilibrium. The explanation, at face value, is based on the distributional character of peer monitoring, where cooperative members primarily seek to shift the cooperative fine to others.

The paper is structured as follows. Section 2 sketches our model. In section 3 we present the centralized management framework where we propose an individual water use-based incentive scheme. In section 4 we propose the team based-incentive schemes. We state a number of propositions describing the dependence of water overuse on a number of determinants, some of which are themselves determined by more fundamental factors including costs of monitoring. In section 5 we extend the basic analysis to deal with two extensions. Section 6 proposes some policy recommendations to help decision makers tackle the severe groundwater over-exploitation particularly in a fragile environment like the MENA region. Section 7 concludes.

2. The Problem

Consider two identical farmers who produce a homogeneous farm good using water as an input. Suppose that the yield (y) response to water (q) can be described by the relation $y = g(q)$; where $g(\cdot)$ is increasing and concave. Moreover, for technical tractability we assume that $g^{(4)} < 0$ in addition to $g''' > 0$. The cost incurred by each farmer for using water, measured in units of output, is c per unit of water. It may represent the cost of delivering water from the gates of an irrigation canal to the farmer's field, or it may measure the cost of pumping water from an underlying aquifer. In addition, the farmer pays a linear price t per unit of water, a price which is determined by the WA. The profit-maximizing quantity of water equates the marginal value product of water to the marginal cost of generating such a quantity.

$$q : g'(q) = c = t. \tag{1}$$

In the absence of asymmetric information, and abstracting from any shadow cost of public funds that might imply Ramsey-pricing considerations, the WA will wish to set t equal to γ , which represents the full public cost of resource provision, including operation and maintenance (O&M) costs, investment costs, and any shadow cost associated with the scarcity of water, such as extraction externalities associated with pumping from a shared aquifer.

When the farmer's water use is his/her private information (unlike the total amount of water use by farmers which is observable to the WA), the farmer is allocated a fixed quota denoted by \bar{q} . This quota is based on the criterion that the rate of utilization of the resource must not exceed the rate of replenishment². However, the farmer who is equipped with an individual water meter to indicate the true intake can well exceed the allocated quota by manipulating the meter. We write the amount of water used in excess (also referred to as the amount of water stolen) as $a = q - \bar{q}$.

The response of the WA will differ according to whether it uses an incentive scheme based on an individual water use, namely the centralized water management, or it resorts to total water use-based incentive schemes. For the latter situation, two schemes are proposed. The first one corresponds to the framework of moral hazard in team problems where the payment for water by the team members is based on the team, rather than on the individual's water use. In the second scheme, the WA makes use of the informational advantages farmers have over the WA because of their long standing and high trade links (especially in close-knit society). This is possible through the implementation of cooperatives characterized by a collective responsibility rule, which makes all farmers jointly responsible for aggregate quantities of water used. This feature is likely to induce peer monitoring by cooperative members, which is possibly a more efficient way of reducing theft than the central monitoring applied by more centralized management structures.

3. The Centralized Scheme

The centralized mechanism is described as follows:

The WA invests in monitoring devices that make water intakes observable. Monitoring incurs a social cost denoted by $\Psi(m)$, which is increasing and convex. The cost of monitoring includes not only measurement devices but also other costs such as the wages of monitors.

If the farmer is not monitored, then he pays the mandated water fee associated with his allotment, $t\bar{q}$. Otherwise, he is discovered stealing with a probability $P(m)$ which increases in the intensity of monitoring. To simplify the exposition the probability $P(\cdot)$ is assumed to be commonly known and takes the form

$$P(m) = \min\{\kappa m, 1\}, \quad (2)$$

where $\kappa > 0$ (we assume henceforth that it is sufficiently small to generate an interior solution³).

² This is captured by constraint (C2) in the optimization problem of the WA, which will be discussed in what follows.

³ The quantity κm cannot be greater than 1 for the following reasons. The centralized structure monitoring the behavior of each individual farmer is very costly, especially when the number of farmers operating in the irrigated area is large (for instance in Tunisia, government agencies manage public irrigated areas with more than four hundred farmers). This essentially implies that monitoring cannot be high enough (otherwise, the WA could prevent theft completely). Therefore, we can always choose a parameter κ sufficiently small to ensure that $\kappa m < 1$, which can be set as the probability of catching a farmer stealing.

When the farmer is discovered exceeding his allocated quota, his true intake is established without error and he pays $t\bar{q}$ plus a penalty proportional to the amount of water stolen, F^{cs4} . It is the nature of the monitoring system that makes it possible to use a punishment device based on individual levels of theft. The punishment is a monetary transfer from the farmer to the WA and takes the form

$$F^{cs} = f \max \{a, 0\}, \quad (3)$$

where the punishment rate f is positive and given outside the model⁵.

Finally, for technical tractability we add the following assumption

$$\left\{ (q^f - \bar{q}) + \frac{f^2}{8} \frac{g''(q^f)}{[g''(q^f)]^3} \right\} < 0, \quad (4)$$

where q^f is the quantity of water used by the farmer when he uses the resource freely (i.e., the resource is not regulated). q^f satisfies the following equation, $g'(q^f) = c$.

The order of events is that the WA sets the monitoring level, m , and the water quota or allotment, \bar{q} ; then each farmer chooses the quantity of water to use q^{cs} . In what follows we focus on the subgame perfect equilibrium and solve the model by backward induction. In stage 2 of the game, the farmer chooses q^{cs} in order to maximize his expected payoff, i.e.,

$$\max_q U^{cs}(q; m)g(q) - cq - t\bar{q} - \kappa mf(q - \bar{q}).$$

Whose first-order condition is

$$g'(q^{cs}) = c + \kappa mf, \quad (5)$$

Clearly, an increase in the levels of monitoring and punishment reduces the required input level, improving incentives for efficient water use. Moreover, when monitoring is poor, i.e., if $m < \frac{t}{\kappa f}$, there is an increased distortion⁶ of water use with respect to the full-information

case. This is because inequality $m < \frac{t}{\kappa f}$ means that overusing the resource is beneficial for the farmer, i.e., the expected benefit from overusing water is positive, $[t - \kappa mf](q^{cs} - \bar{q}) > 0$.

Now let us turn to the initial contracting stage, where the WA anticipates the farmer's behavior and picks a level of monitoring, m , and a water allotment \bar{q} (which is equal for all farmers in the area) that maximize the social benefit. Specifically this benefit function is the sum of the farmers' surpluses $2[g(q) - cq - t\bar{q} - \kappa mf(q - \bar{q})]$ and the water supplier's surplus

⁴ The superscript "cs" is to indicate the centralized structure by contrast to "c" which will be used for cooperatives.

⁵ We are not concerned here with the optimal choice of f since our focus is on water management incentive schemes.

⁶ When $m < \frac{t}{\kappa f}$, we have $c + \kappa mf < c + t$, which implies that $g'(q^{cs}) < g'(q^{fi})$ (where the superscript fi refers to the full-information case.) Since the function g is concave (its first derivative function, g' is decreasing in its argument) then, one gets $q^{cs} > q^{fi}$.

which is equal to the revenue from the expected payment from water use, $2[t\bar{q} + \kappa mf(q - \bar{q})]$, from which is deduced the cost of water provision to the irrigated area, $2\gamma q$, and the cost incurred by monitoring, $2\psi(m)$.

$$W^{cs}(m, \bar{q}) = 2[g(q^{cs}) - (c + \gamma)q^{cs} - \psi(m) - \Phi(f)].$$

This equation states that the WA trades off efficiency gains induced by monitoring against monitoring costs. The WA must also consider two major constraints. The first is

$$2q \leq \tilde{Q}. \quad (C1)$$

where \tilde{Q} is the quantity of water available or also the storage capacity or the stock of the resource *in situ* in the aquifer. This constraint reflects the scarcity of the resource (farmers can use what is available at most). The second constraint is

$$2\bar{q} = \tilde{Q} - Q_{sec}. \quad (C2)$$

where the quantity Q_{sec} is a security stock of water ($Q_{sec} < \tilde{Q}$). This constraint reflects that the rate of utilization of the resource must be lower than the rate of replenishment to prevent the deterioration of the aquifer.

In what follows, proposition 1 characterizes the solution to the WA's problem where the WA maximizes its objective function $W^{cs}(m, \bar{q})$ with respect to m and \bar{q} taking into account the two constraints (C1) and (C2).

Proposition 1:

The optimal policy used by the $WA(m^{cs}, \bar{q})$ satisfies

$$m^{cs} : h \frac{\kappa f^{cs}}{g''(q^{cs})} = \psi'(m^{cs}), \quad (6)$$

and

$$\bar{q} = \frac{\tilde{Q} - Q_{sec}}{2}. \quad (7)$$

Where μ is the Lagrangian multiplier on the water availability constraint (C1) and $h = [\kappa m^{cs} f - \gamma - \mu]$.

(For the proof, refer to the Appendix)

The proposition says that some monitoring is always required in equilibrium. However, because monitoring is costly, the optimal response of the WA is to tolerate some water overuse in order to save on monitoring costs. Moreover, the monitoring level responds directly to the scarcity of the resource, captured by the parameter μ (which can be interpreted as the scarcity rent of water).

$$\frac{\partial m^{cs}}{\partial \mu} = \frac{\kappa f}{g''(q^{cs})} \frac{1}{\left[\frac{(\kappa f)^2}{g''(q^{cs})} - [\kappa m f - \gamma - \mu] \frac{g'''(q^{cs})}{g''(q^{cs})} - \psi''(m^{cs}) \right]} > 0. \quad (8)$$

This implies that the more severe the shortage of water, the higher the required monitoring effort to reduce the overuse of the resource.

4. Team-Based Incentive Schemes

4.1 First scheme

We assume that the total amount of water used by the two farmers, $Q = q_1 + q_2$, is publicly known without any cost, and can be contracted for directly. In particular, the WA designs a team-based incentive scheme where it asks the farmer to pay the fixed water fee associated with his/her allocated quota, $t\bar{q}$, and a share of the full extra amount if actual water use exceeds the total quota allocated to the group, $s_i[t.(Q - 2\bar{q})]$ for $i = 1, 2$, where $s_i(.)$ is differentiable. Since farmers are identical then, we can assume that each farmer has the same share from the total liability, $t.(Q - 2\bar{q})$ i.e., $s_i[t.(Q - 2\bar{q})] = s[t.(Q - 2\bar{q})]$ for $i = 1, 2$.

The order of events is that the WA sets the price of water, t , and then each farmer chooses the quantity of water to use⁷ q^{nbb} that maximizes his expected payoff

$$\max_q U(q) = g(q) - cq - t\bar{q} - s[t.(Q - 2\bar{q})].$$

where the first-order condition is

$$g'(q) = c + s'[t.(Q - 2\bar{q})].t, \tag{9}$$

Comparing equations (1) and (9) yields

$$s'[t.(Q - 2\bar{q})].t = t,$$

and thereby

$$s'[t.(Q - 2\bar{q})] = 1,$$

implying that each farmer has to pay a total liability, $t.(Q - 2\bar{q})$, to the WA

$$s[t.(Q - 2\bar{q})] = t(Q - 2\bar{q}). \tag{10}$$

Therefore, this mechanism encourages the farmer to use the full-information water use level

$$g'(q) = c + t. \tag{11}$$

The WA (the principal) can restore efficiency by administering incentive schemes that do not balance the budget⁸ since both farmers will be paying the full extra amount, $t(Q - 2\bar{q})$. It is worth noting that the above incentive scheme works independently of the team size, but it may be infeasible when there are endowment constraints. This is why one may resort to an alternative group incentive scheme that does not violate individual endowment constraints, namely cooperatives.

4.2 Cooperative structure

The cooperative is characterized by a collective responsibility rule described as follows: if theft occurs, the cooperative as a whole receives a punishment proportional to the total amount of water used in excess (which could also be referred to as the total amount of water stolen):

⁷ The superscript "nbb" is to refer to the non-balanced budget setting.

⁸ If the principal has instead administered the incentive scheme when the total liability $t(Q - 2\bar{q})$, was fully shared among the agents, this would result in an inefficient outcome (see Holmstrom, 1982). The point is therefore not that group punishments are the only effective scheme, but rather budget-breaking is the essential instrument in neutralizing the free-riding problem.

$$F^c = f\left(\sum_{i=1,2} q_i - 2\bar{q}\right). \quad (12)$$

Now suppose that, relative to the WA, farmers have a comparative advantage in monitoring each other because of geographical proximity and high trade links. We assume that peer monitoring brings about only evidence of the occurrence of water overuse (or water theft) but not of its amount. The WA may then contemplate the possibility of inducing peer monitoring between the two farmers, typically through the establishment of a cooperative governed by rules that make all group members jointly liable. If theft occurs in the cooperative, the fine is shared equally between farmers who are caught overusing the resource, otherwise it is shared by all members.

Peer monitoring incurs a private cost $\psi(m)$ to the farmer, assumed to be increasing and convex. Each member commits to a level of monitoring⁹ (observable by other members) before actual water use is decided. The probability that a farmer i is caught overusing water is therefore given by:

$$P_i(m_j) = \min\{\kappa m_j, 1\}, \quad (13)$$

where $\kappa > 0$ (which is sufficiently small to ensure an interior solution). This probability increases in the monitoring level of the other. Farmers do not collude in either their monitoring or their production decisions¹⁰.

The order of events is therefore that the WA fixes t and the farmer's allotment \bar{q} , then individual members choose m_i , then having observed each others' choice of m_i , they choose the amount of water they want to use q_i .

Notice that the outlined cooperative framework implies that in the absence of any results of their monitoring efforts, farmers would share the cooperative fine equally. By monitoring each other, farmers reallocate the burden of the fine between themselves. Put differently, peer monitoring determines the ex post shares of the fine for everyone as well as the size of the fine. Denote by $s_i^{\text{exp}}(m_i, m_j)$ the farmer i 's expected share of such a fine. Suppose, first, that only farmer i overuses water, i.e., $a_i > 0$ and $a_j \leq 0$; then the distribution of the fine is determined solely by the monitoring applied by farmer j , m_j ; which increases the expected share of farmer i and decreases that of his peer, farmer j .¹¹

$$\begin{cases} s_i^{\text{exp}} = \frac{1}{2}(1 + \kappa m_j) \\ s_j^{\text{exp}} = \frac{1}{2}(1 - \kappa m_j) \end{cases}, \quad (14)$$

⁹ One may think of observable sunk investments (such as tools and equipment) made by members of the cooperative, which would commit them to a higher monitoring intensity. For instance, it is widely observed in countries like Tunisia that landlords build little houses in their farms where they can keep some farm equipment for daily use and where both landowners and agricultural laborers may spend some time.

¹⁰ For the moment we sidestep the issue of collusion in monitoring efforts and we tackle it later on in the paper.

¹¹ The expected share of farmer i from the cooperative fine is given by $s_i^{\text{exp}} = \kappa m_j + \frac{1}{2}(1 - \kappa m_j)$ where the first and second term correspond respectively to his share when he is caught and when not. Analogously, farmer j 's expected share is given by $s_j^{\text{exp}} = (1 - \kappa m_j) + \frac{1}{2}\kappa m_j$

Now suppose that both farmers overuse water, i.e., $a_k > 0$ for $k = i, j$. The expected share¹² of farmer i is lowered by the monitoring effort applied by his peer, and is in turn increased by his own monitoring effort.

$$s_i^{\text{exp}} = \frac{1}{2}(1 - \kappa m_i + \kappa m_j). \quad (15)$$

The subgame perfect equilibrium corresponds to the profile $(m_1^c, m_2^c, q_1^c, q_2^c)$ of monitoring efforts $m_i^c \geq 0$, water use levels $q_i^c : [0, +\infty)^2 \rightarrow [0, \tilde{Q})$ mapping from the set of monitoring decisions into the set of water use decisions. In what follows we shall focus on symmetric equilibria which imply that

$$q_i^c = q^c \text{ and } m_i^c = m^c \text{ for } i=1,2.$$

The solution to this problem is summarized by proposition 2.

Proposition 2:

There exists a unique symmetric subgame perfect equilibrium such that $(m^c; q^c)$ satisfies

$$q^c : g'(q) = c + \frac{1}{2}f, \quad (16)$$

$$m^c = \phi(k_2). \quad (17)$$

Where, $k_2 = f \left[(q^c - \bar{q}) - \frac{1}{4} \frac{f}{g''(q^c)} \right]$ and $\phi = (\psi')^{-1}$.

(For the proof, refer to the Appendix)

Peer monitoring has a purely distributional effect—it may allow every cooperative member to shift the cooperative fine on the others. The immediate corollary from the above proposition is that the cooperative outcome is inefficient when the punishment rate f is different from twice the price of water, $2t$. The direction of distortion depends on the severity of the punishment rate. If it is sufficiently high (i.e. when f lies above $2t$) there will be a downward distortion of the cooperative water use with respect to the efficient level. Otherwise, (i.e. when f is lower than $2t$) the distortion will be upward. This is summarized as follows:

Corollary 1:

If $f > 2t$, then $q^c < q^{\text{nbbs}}$.

If $f < 2t$, then $q^c > q^{\text{nbbs}}$.

As one can see, with a sufficiently stringent punishment, cooperatives preserve the resource more than the full-information setting. In this problem the preservation of the resource should be the primary objective of any policy in the short run, since the exhaustion of the resource constitutes a real threat when the rate of utilization exceeds the rate of replenishment. This

¹² The expected share of farmer i from the cooperative fine when everyone overuses water is given by

$$s_i^{\text{exp}} = \frac{1}{2} \kappa m_i \kappa m_j + \kappa m_j (1 - \kappa m_i) + \frac{1}{2} (1 - \kappa m_i) (1 - \kappa m_j)$$

Where the first term corresponds to his share when both farmers are caught overusing the resource, the second term is his share when he is caught and farmer j is not, and the last term is his share when none is caught.

objective seems to conflict with social welfare considerations in the short run, however this is not the case in the long-run perspective since the two objectives coincide.

4.3 Comparative statics

This section evaluates the effect of changes in the model's key parameters on the incentives of theft and monitoring in equilibrium. To obtain explicit solutions where possible, we assume that monitoring costs take the quadratic form $\psi(m) = \frac{1}{2}bm^2$ where $b > 0$. We start by examining the impact of the level of punishment f on the equilibrium amount of water used in excess, which is found to be decreasing because it reduces the net returns to water overuse.

$$\frac{\partial(q^c - \bar{q})}{\partial f} = \frac{\partial q^c}{\partial f} = \frac{1}{2g''(q^c)} < 0 \quad (18)$$

We now show how the intensity of monitoring will vary with the costs of monitoring, and punishment levels. From equations (15) and (16) above, we obtain the following comparative static results. First, the monitoring effort is decreasing in the costs of monitoring. The explanation is straightforward.

$$\frac{\partial m^c}{\partial b} = -\frac{k_2}{b^2} < 0. \quad (19)$$

Second, monitoring is decreasing in the price of water and in the level of monitoring.

$$\frac{\partial m^c}{\partial f} = \frac{1}{b} \left[(q^c - \bar{q}) + \frac{f^2}{8} \frac{g'''(q^c)}{[g''(q^c)]^3} \right] < 0 \quad (20)$$

The explanation is that an increase in the level of punishment, f , reduces the incentives for water overuse lowering thereby the expected punishment burden, and thus requiring a lower monitoring effort to reduce it.

It is always interesting to compare the equilibrium peer monitoring effort to the socially optimal one. For this purpose, we consider the second-best problem faced by the WA as a social planner who can control monitoring efforts of farmers but not their water usage choices once monitoring decisions have been made. Moreover, assume that the WA cannot affect the incentives of water overuse for given monitoring efforts. In particular, the WA cannot ensure that farmers do not overuse the resource. The WA picks a monitoring effort, m^* , that maximizes the following social welfare function

$$\max_{m \geq 0} 2[g(q^c) - (C + \gamma)q^c - \psi(m)] \quad (21)$$

It is socially optimal not to monitor in cooperatives governed by these rules whenever monitoring is costly: that is $m^* = 0$. Consequently, farmers over-monitor in equilibrium, $m^c > m^*$; because of a rent seeking behavior effect. Since peer monitoring has a purely distributional purpose, each cooperative member is ready to bear the costs of monitoring in order to get higher rents from overusing water whenever he succeeds to shift the total cooperative fine on the others.

4.4 Endogenous punishment

Here we extend the model to the punishment rate f to be chosen collectively by cooperative members at an initial contracting stage, subject to a cost of inflicting punishment $\varphi(f)$ which is increasing and sufficiently convex to ensure an interior solution. This cost may be pecuniary or may correspond to costs in the deterioration of social relations that occur when punishment is inflicted on members of a close-knit society. Members choose the punishment

level f^c that maximizes an objective function defined as the sum of cooperative members' surpluses: $2 \left[g(q^c) - cq^c - t\bar{q} - fa^c - \frac{1}{2}b(m^c)^2 \right] - \varphi(f)$ plus the surplus of the WA, which is equal to its revenue from water proceeds, $2t\bar{q}$, from which is deducted the cost of supplying water to the cooperative area, $2\gamma q^c$.

$$\max_f W^c(f) = 2 \left[g(q^c) - (c + \gamma)q^c - fa^c - \frac{1}{2}b(m^c)^2 \right] - \varphi(f) \quad (22)$$

This has a first-order condition:

$$f^c : -\frac{1}{g''(q^c)} \left[\gamma + \frac{1}{2}f \right] - 2(q^c - \bar{q}) - 2\frac{k_2}{b} \left\{ \left[(q^c - \bar{q}) + \frac{f^2}{8} \frac{g'''(q^c)}{[g''(q^c)]^3} \right] \right\} = \varphi'(f) \quad (23)$$

From this condition, one can show that the punishment level is increasing in monitoring costs. Totally differentiating the first order condition with respect to f and b and rearranging yields:

$$\frac{\partial f^c}{\partial b} = \frac{2k_2}{Gb^2} \left\{ \left[(q^c - \bar{q}) + \frac{f^2}{8} \frac{g'''(q^c)}{[g''(q^c)]^3} \right] \right\} > 0. \quad (24)$$

Where $G = \frac{\partial^2 W^c}{\partial f^2} < 0$. This result confirms that the two instruments, monitoring and punishment are indeed substitutes, since an increase in the cost of one can be compensated for by an increase in the level of the other.

4.5 Cooperative size

The analysis thus far has remained restricted to the two-farmer cooperative. In practice, however, most cooperatives where irrigation water is based on aquifers involve up to as many as 40 farmers, and most involve more than 100 farmers when irrigation is based on surface methods. In this section we address the issue of optimal group size, where the basic setup is extended from the two-farmer cooperative to an n -farmer one.

We characterize the symmetric subgame perfect equilibrium (q_n^c, m_n^c) (assuming that the second-order condition for a maximum holds¹³):

$$q_n^c : g'(q) = c + \frac{1}{n}f, \quad (25)$$

$$m_n^c : k_n \Phi_n(\kappa m) = (n-1)\psi'[(n-1)m]. \quad (26)$$

Where, $k_n = f \left[(q_n^c - \bar{q}) - \frac{1}{n^2} \frac{f}{g''(q_n^c)} \right] > 0$ and

$$\Phi_n(\kappa m) = (1 - \kappa m)^{(n-2)(n-1)} \sum_{k=1}^{n-1} (1 - \kappa m)^{(k-1)} = \frac{1}{\kappa m} (1 - \kappa m)^{(n-2)(n-1)} [1 - (1 - \kappa m)^{n-1}]$$

¹³ It is quite difficult to derive the second-order condition for this problem because the first-order conditions account for the highly complicated term

$$\phi(\kappa m) = (1 - \kappa m)^{(n-2)} \sum_{k=1}^{n-1} (1 - \kappa m)^{(n-1)(k-1)}.$$

From the necessary conditions one can see that the farmer increases his/her water use as the cooperative becomes larger, i.e.,

$$\frac{\partial q_n^c}{\partial n} = 1 - \frac{1}{n^2} \frac{f}{g''(q_n^c)} > 0. \quad (27)$$

(meaning that larger groups increase the incentives for overusing the resource). However, it is not clear whether the equilibrium monitoring intensity, m_n^c tends to increase or decrease in the cooperative size. The intuition suggests that the group size affects the incentive problem in two ways. A larger group discourages individual monitoring, since the evidence of the farmer's theft can be established if he/she is discovered stealing by at least one of their peers. Monitoring altogether might then become useless for peer farmers since the same outcome can be achieved with a smaller number of them, avoiding thereby the useless duplication of monitoring. This free-riding problem lowers the cooperative members' incentives for monitoring.

On the other hand, a larger group may increase the total amount of theft in the cooperative increasing thereby the maximum punishment that would be incurred by a member who was the only one caught. This acts as an incentive in the opposite direction and increases the member's incentives to monitor more to catch others exceeding their quotas, which may then reduce his/her expected share of the total fine. This rent-increasing effect will thus counteract the above free-riding effect by encouraging more intense monitoring as the number of farmers in the group increases.

The major difficulty for determining the level of monitoring in equilibrium is that this variable is implicitly determined by equation (25). In order to get some insights, we will proceed in the remainder of the section to the following simplification: we restrict attention to sufficiently small values of κ which implies that all terms in κ^n for $n \geq 2$ become of the second order and can thereby be dropped from our calculations. Consequently, when κ is sufficiently small, the first-order condition (25) above reduces to¹⁴

$$m_n^c: (n-1)[\psi(n-1)m] \simeq k_n(n-1)[1 - (n-1)(n-2)\kappa m]. \quad (28)$$

To obtain explicit solutions where possible, we assume that monitoring costs take the quadratic form $\psi(m) = \frac{1}{2}bm^2$ where $b > 0$. By rewriting equation (28) we obtain the approximated equilibrium level of monitoring

$$m_n^c \simeq \frac{k_n}{(n-1)[b + k_n\kappa(n-2)]} \quad (29)$$

The equilibrium monitoring effort decreases in b

$$\frac{\partial m_n^c}{\partial b} = -\frac{k_n}{(n-1)[b + k_n\kappa(n-2)]^2} < 0; \quad (30)$$

As for the impact of the cooperative size on monitoring, it is given by

$$\frac{\partial m_n^c}{\partial n} \simeq \theta \frac{\partial k_n}{\partial n}, \quad (31)$$

¹⁴ This simplification involves no major loss of insights, since the simplified expressions of the first-order conditions capture the main qualitative aspects of the solution to the cooperative model.

Where $\theta = \frac{b}{(n-1)[b+k_n\kappa(n-2)]^2} > 0$. This essentially implies that the sign of $\frac{\partial m_n^c}{\partial n}$ is equal to the sign of $\frac{\partial k_n}{\partial n}$. The expression of $\frac{\partial k_n}{\partial n}$ is given by

$$\frac{\partial k_n}{\partial n} = f \left\{ \left(1 + \frac{g'''(q_n^c)}{n^2[g''(q_n^c)]^2} \right) \frac{\partial q_n^c}{\partial n} + \frac{2f}{ng''(q_n^c)} \right\}. \quad (32)$$

By replacing $\frac{\partial q_n^c}{\partial n}$ by its value given by equation (27) into equation (32), one gets the expression of $\frac{\partial k_n}{\partial n}$

$$\frac{\partial k_n}{\partial n} = \frac{f^2}{ng''(q_n^c)} \left[\left(2 - \frac{1}{n} \right) - \frac{g^{(3)}(q^c)}{n^3[g''(q_n^c)]^2} \right]. \quad (33)$$

This partial derivative has an ambiguous sign because $\frac{f^2}{ng''(q_n^c)} < 0, (2 - \frac{1}{n}) > 0$ and $-\frac{g^{(3)}(q^c)}{n^3[g''(q_n^c)]^2} < 0$. This essentially implies that the sign of $\frac{\partial m_n^c}{\partial b}$ is ambiguous. Because of the analytical complexity, we will resort to the use of a numerical example to answer this question. The example is:

the production function $g(q) = \sqrt{q}$;

the per-unit private cost and price of water are $c = t = 0.2$;

the transaction costs related to monitoring takes two different values $b = 3$ and $b = 10$.

Simulations suggest the shape and the value of the monitoring effort, $m(n)$ as a function of the group size considerably changes when the costs of monitoring vary. When $b = 3$ the monitoring function is gradually decreasing when the cooperative becomes larger, i.e., for $n \geq 3$. This means that the free riding effect always tends to dominate for sufficiently small monitoring costs. In contrast, when $b = 10$ the monitoring levels become smaller and, what is more important, the function is increasing for small values of n and starting from $n = 4$ it gradually decreases. To put it differently, when the monitoring costs are sufficiently large it means that the rent seeking effect might come into play (this is depicted by figure 1).

The results of the simulation suggest that there exists a level \bar{b} of monitoring costs such that

- for any $b < \bar{b}$ the equilibrium monitoring effort $m^c(n)$ is a decreasing function of the cooperative size, n , and
- for any $b > \bar{b}$ the equilibrium level of monitoring $m^c(n)$ increases up to some level \tilde{n} and then gradually declines.

The results obtained above allow us to study the issue of the optimal cooperative size. Farmers may seek a group size n_{max} that maximizes the average cooperative benefit function¹⁵ $W_A^C(n)$

¹⁵ The choice of the average rather than the absolute social welfare function relies on the fact that for the latter, the group size effect may always dominate and the function is likely to be always increasing in the cooperative size.

$$n_{max} \in \arg \max_{n \geq 2} W_A^c(n) = g(q_n^c) - (c + \gamma)q_n^c - \psi(m_n^c) \quad (34)$$

which has the first-order condition for an interior solution (assuming that the second-order condition holds)

$$[g'(q^c) - (c + \gamma)] \left(\frac{\partial q^c}{\partial n} \right) - \psi'(m^c) \left(\frac{\partial m^c}{\partial n} \right) = 0 \quad (35)$$

The (first-order) change in social welfare attributable to a marginal entrant (=a new cooperative member) is composed of two terms. The first term implies that the new entrant causes every member to free ride on his/her peers and thus to contract his/her monitoring effort. This would provide more opportunities for overusing water for everyone. This stealing effect causes a reduction in social welfare of

$$[g'(q^c) - (c + \gamma)] \left(\frac{\partial q^c}{\partial n} \right) < 0, \quad (36)$$

On the positive side, the free riding of farmers on each other's monitoring efforts allows them to save on their monitoring costs. This cost-saving effect generates an increase in social welfare of

$$- \psi'(m^c) \left(\frac{\partial m^c}{\partial n} \right) > 0 \quad (37)$$

The optimal cooperative size, n_{max} , thus equates the social marginal benefit stemming from the additional savings in monitoring costs to the social marginal losses caused by a higher occurrence of theft. This implies that the net benefits of peer monitoring are maximized when the size of the cooperative is neither too small (due to the "monitoring cost savings" effect) nor too large (due to the "stealing" effect).

The effect of varying the group size on the cooperative welfare is found to be analytically complicated, that is why we use the same numerical example (and where $\gamma = 3$) which sheds light on the intensity of stealing and cost-saving effects when one varies monitoring costs. Results of the simulations show the behavior of the welfare function for $b = 3$ and $b = 10$. We find that when $b = 3$, the welfare attains a maximum at the point $n_{max} = 4$, while it is gradually decreasing in the other. This points to the fact that the stealing effect dominates almost everywhere and the best policy for the WA might be to restrict or even to reduce the cooperative size. Put differently, it is socially desirable to have small cooperatives.

5. Extensions

5.1 Collusion

The cooperative model described above corresponds to a non-cooperative game. Each cooperative member is out to maximize his expected payoff, and makes his monitoring effort and water input levels decisions independently of the other members. What happens if we relax this assumption and consider possibilities of coordinated actions for monitoring efforts?¹⁶

Naturally, the model to consider is what happens if two cooperative members choose their monitoring efforts in order to maximize joint payoffs, $[U_i(q_i, m_i) + U_j(q_j, m_j)]$. The collusive outcome is summarized by the following corollary:

Corollary 2:

The coordinated or collusive monitoring efforts are

¹⁶ It is easier to consider collusion in monitoring efforts (unlike collusion in individual amounts of water use which are the farmers' private information and cannot be observed by their peers).

$$m_i = m_j = 0. \quad (38)$$

(For the proof, refer to the Appendix)

The collusive monitoring effort is efficient. This result is quite intuitive. Before the occurrence of collusion, cooperative members compete in monitoring because of the rent-seeking effect, even though their monitoring is useless since they always equally share in the cooperative fine. Collusion is thus socially beneficial since it realizes the same outcome, and results in substantial monitoring cost-savings.

5.2 Monitoring structures

Although in practice the mutual monitoring structure—whereby each farmer in the group is being simultaneously monitored by all of her peers—is commonly observed, other monitoring structures deserve consideration. An interesting departure from the mutual structure is the “rotating monitoring”¹⁷ structure in which every farmer monitors only one of his peers, say his left neighbor, and is in turn monitored by his right neighbor. There is a natural argument in favor of monitoring structures of the latter kind, namely the duplication in the mutual structure, which obviously takes place when the number of cooperative members exceeds two. As a first and very tentative attempt to explore the issue of the optimal design of peer monitoring structures, we compare, in this section, between the mutual (MU) structure and the rotating monitoring (RM) structure with regard to the equilibrium water use and the equilibrium monitoring efforts and thereby to the cooperative welfare level. The comparison will be held in the context of a three-farmer cooperative.

5.2.1 Mutual monitoring structure

Consider a cooperative formed by three farmers i , j and k . In this cooperative, members apply mutual peer monitoring whereby each farmer monitors all his/her peers. We assume that each farmer applies equal monitoring efforts to monitor all his peers, which implies that the total cost of monitoring applied by a farmer, say farmer i , is equal to $\psi(2m_i)$. The joint-responsibility clause states that a farmer pays one-third of the cooperative fine in either case, all farmers are caught stealing (exceeding their allocated quotas) or none is caught. The farmer bears half of the fine if he is caught and one of his peers is also caught. He bears the whole fine if he is the only one caught and pays nothing if he is not caught and all his peers are.

Let ρ_i^C and ρ_i^N denote the probabilities of the events when farmer i is caught/not caught stealing the resource, which are defined respectively by:

$$\rho_i^C = 1 - \rho_i^N = p_{ij} + p_{ik} - p_{ij}p_{ik}, \quad (39)$$

$$\rho_i^N = (1 - p_{ij})(1 - p_{ik}) \quad (40)$$

where p_{ij} and p_{ik} are the probabilities that farmer i is caught by farmer j and farmer k respectively. p_{ij} and p_{ik} are proportional to the monitoring efforts performed by farmers j and k

$$p_{ij} = \kappa m_j \text{ and } p_{ik} = \kappa m_k \quad (41)$$

By the same token, p_{ji} and p_{jk} are the probabilities that farmer j is caught by farmer i and farmer k respectively.

¹⁷ The notion of "rotating monitoring" here differs from that used by Armandariz DeAghion (1999).

$$p_{ji} = \kappa m_i \text{ and } p_{jk} = \kappa m_k \quad (42)$$

And p_{ki} and p_{kj} are the probabilities that farmer k is caught by farmer i and farmer j respectively.

$$p_{ki} = \kappa m_i \text{ and } p_{kj} = \kappa m_j \quad (43)$$

Taking into account the punishment sharing rule and the fact that the events of catching farmer i , farmer j and farmer k are independent, the expected share of farmer i from the total fine, denoted by S_i^3 is equal to

$$S_i^3 = \frac{1}{3} \left\{ \rho_j^N \rho_k^N + \rho_i^C \left[1 + \frac{1}{2} (\rho_j^N + \rho_k^N) \right] \right\}, \quad (44)$$

where,

$$\rho_i^C = \kappa m_j + \kappa m_k - k^2 m_j m_k,$$

and

$$\rho_i^N = (1 - \kappa m_j)(1 - \kappa m_k)$$

By replacing ρ_i^C and ρ_i^N by their expressions into the expression of S_i^3 , one gets S_i^3 as a function of the monitoring efforts, m_i , m_j , and m_k

$$\begin{aligned} S_i^3(m_i, m_j, m_k) &= \frac{1}{3} \left\{ (1 - \kappa m_i)^2 (1 - \kappa m_j)(1 - \kappa m_k) + (\kappa m_j + \kappa m_k \right. \\ &\quad \left. - k^2 m_j m_k) \left(1 + \frac{1}{2} [(1 - \kappa m_i)(1 - \kappa m_k) + (1 - \kappa m_i)(1 - \kappa m_j)] \right) \right\} \end{aligned} \quad (45)$$

We characterize the symmetric subgame perfect equilibrium where $q_i^c = q_j^c = q_k^c = q_3^c$ and $m_i^c = m_j^c = m_k^c = m_3^c$ (assuming that the second-order condition for a maximum holds)¹⁸

$$q_3^c: g'(q) = c + \frac{1}{3} f \quad (46)$$

The equilibrium monitoring effort satisfies the following implicit equation

$$m_3^{c(M)}: -\frac{f^2}{9g''(q^c)} \left[\kappa^4 m^4 + \frac{1}{2} \kappa^4 m^3 - 2\kappa^3 m^3 - \frac{5}{2} \kappa^3 m^2 + 3\kappa^3 m \right] - f(q^c - \bar{q}) [\kappa^4 m^3 - 3\kappa^3 m^2 + 4\kappa^2 m] \quad (47)$$

For a sufficiently small parameter, κ , the terms κ^n for $n \geq 2$, become of the second order and the above equation reduces to

$$\kappa f(q^c - \bar{q}) \simeq \psi(2m^{c(M)}). \quad (48)$$

5.2.2 Rotating monitoring structure

Consider the same framework of a three-farmer cooperative formed by farmers i, j and k . In this cooperative, members apply rotating monitoring whereby each farmer monitors only his left neighbor. We assume that farmer i monitors farmer j , with the monitoring effort $m_{ij} = m_i$, and farmer j monitors farmer k , with the monitoring effort $m_{jk} = m_j$ and farmer k monitors farmer i , with the monitoring effort $m_{ki} = m_k$. The joint-responsibility clause is the same as in the previous section.

¹⁸ See the proof in the Appendix, (D. Mutual monitoring).

Let $p_i = \kappa m_k$, $p_j = \kappa m_i$ and $p_k = \kappa m_j$ be the probabilities that farmers i , j and k are respectively caught. Taking into account the punishment sharing rule and the fact that the events of catching farmer i , farmer j and farmer k are independent, the expected share for farmer i from the total fine, S_i^3 is

$$S_i^3 = \frac{1}{3} \left\{ p_i p_j p_k + (1 - p_j) + \frac{3}{2} [p_i p_j (1 - p_k) + p_i p_k (1 - p_j)] + 3 p_i (1 - p_j) \right\}, \quad (49)$$

By replacing p_l for $l = i, j$ and k by their expressions, we obtain S_i^3 as a function of the monitoring efforts, m_i , m_j and m_k as follows

$$S_i^3(m_i, m_j, m_k) = \frac{1}{3} \left\{ \begin{aligned} &\kappa^3 m_i m_j m_k + (1 - \kappa m_k)(1 - \kappa m_j)(1 - \kappa m_i) \\ &+ \frac{3}{2} [\kappa^2 m_k m_i (1 - \kappa m_j) + \kappa^2 m_k m_j (1 - \kappa m_i)] \\ &+ 3 \kappa m_k (1 - \kappa m_i)(1 - \kappa m_j) \end{aligned} \right\} \quad (50)$$

We characterize the symmetric subgame perfect equilibrium where $q_i^c = q_j^c = q_k^c = q_3^c$ and $m_i^c = m_j^c = m_k^c = m_3^c$ (assuming that the second-order condition for a maximum holds)¹⁹

$$q_3^c: g'(q) = c + \frac{1}{3}f, \quad (51)$$

The equilibrium monitoring effort satisfies the following implicit equation

$$m^{c(R)}: \left(-\frac{1}{2} \kappa^2 m + \kappa \right) \left[-\frac{f^2}{9g''(q^c)} + f(q^c - \bar{q}) \right] = \psi'(m). \quad (52)$$

For a sufficiently small parameter, κ , the terms κ^n for $n \geq 2$, become of the second order and the above equation reduces to

$$\left[-\frac{\kappa f^2}{9g''(q^c)} + \kappa f(q^c - \bar{q}) \right] \simeq \psi'(m^{c(R)}). \quad (53)$$

Straightforward comparison between equations (48) and (52) shows that the equilibrium rotating monitoring effort is higher than twice the equilibrium mutual monitoring effort

$$m^{c(R)} > 2m^{c(M)} \quad (54)$$

At first glance this result seems to be counterintuitive, but if we look at it more closely we can see that this is not the case. The explanation, at face value, is based on the distributional character of monitoring cooperative members who have a rent seeking behavior, seeking primarily to shift the cooperative fine on the others. In the mutual monitoring structure, each farmer monitors all his peers increasing thereby the possibility of sharing the fine with more than one member even in the case when all others choose to free ride on him by not monitoring.

However, in the rotating monitoring structure, each farmer monitors only one of his neighbors, reducing his chance to reduce his share from the cooperative fine. That is why the farmer monitors more intensively to increase the probability of his neighbor stealing and to at least share the fine with him.

6. Policy Implications

Recognizing that neither centralized management by the state, nor a "laissez faire" market-driven system ensure the sustainable use of renewable ground water resources, this paper has presented a participative management model with built-in incentives for economically more efficient and sustainable aquifer use. Thus the central idea of this research is to design the appropriate institutions and rules to the effective management of groundwater. Indeed our

¹⁹ See the proof in the Appendix, (E. Rotating monitoring).

analysis shows that the conjunction of collective actions and right punishment rates will collude to preserve the resource. This result will certainly help decision makers to promote and implement a sustainable management of this vital resource. Since surface water is not only scarce but also highly uncertain and often of bad quality in the MENA region, groundwater constitutes a unique guarantee, if it is well managed. Its long-term viability is necessary for the survival of the agricultural sector, and thus providing food security for those fragile countries.

Hence, the principal aim of this research is to design the appropriate institutions and rules to achieve the sustainable management of overused ground-water and especially to implement them smoothly with the user's approval. It appears to us that there is a decisive aspect to a feasible approach for groundwater management along the lines we have presented above. Indeed the State needs to support the collective management system by progressively introducing a robust regime of authorizations and dissuasive taxes for groundwater users who are unwilling to join the collective management.

The keys that unlock the way for transferring the groundwater use from a situation of non regulated wells freely pumping the water to a system of well regulated collective exploitation of the resource are "trust in the management" and "well-being of the individual". Trust in a system of collective management comes from adequate participation and representation. Owning a capital stake in the common enterprise needs to be "decisively" understood by well users as a satisfactory position for them providing them with adequate access to all the decision-making. This paramount state of stakeholder access to management should never be subordinated to the more "technical" interests of aquifer management. Put simply, everybody involved (users, managers and government) need to understand that the success of the commonly owned enterprise is the purpose of this vehicle for sustainable water use and everybody must trust that the success of the enterprise will be the means to ensure the preservation of the aquifer.

For the water users, the sensation of "well being" surely comes from being better off having made the decision to participate. Clearly, the important benefit of participating in a system that assures sustainable resource management should not be understated. However, it is very unlikely that this on its own will provide the incentive needed. More immediate benefits to the user need to be promoted. Firstly, a capitalization of the value of the well could be offered, at least partially, literally as capital. Secondly, the enterprise (association) should offer a range of meaningful benefits for the users to boost their 'market' performance. Technical support that aids users to get better value from their "managed" water supply should be at the heart of the programs offered.

The State should play a decisive role in the implementation. The preservation of an economically important aquifer can be worth an enormous amount to a regional economy and to the nation in the long term. Therefore, opening the way for reasoned State investment, providing capital grant incentives and the appropriate capital and banking infrastructure for the enterprises to get established, is justified.

The successful implementation of collective groundwater management ought to render the authorizing and taxing of groundwater abstractions by users who prefer to remain outside officially recognized participatory management a practical proposition. Only a small number of wells will need to be policed. The taxes for abstractions should progressively rise, eventually becoming prohibitive. In the fullness of time this will be the greatest incentive for aquifer users to seek the 'safe haven' of organized management.

With the successful outcome of the approach it will seem odd to future groundwater users that the proposition of letting 'free riders' pump their precious resource with such gay abandon was ever a notion supported by the majority of well owners in the first place.

7. Conclusion

This paper has investigated the design of the appropriate institutions and rules to achieve sustainable groundwater management by reducing the over pumping of the scarce resource. We have done this by designing two total water use-based incentive schemes where the total water use is publicly observable (unlike the individual water use which is the farmer's private information). In the first scheme, the Water Authority administers an incentive scheme that does not balance the budget, restoring thereby water use efficiency. Such scheme works independently of the team size, but it may be infeasible when farmers have endowment constraints. This is why the WA resorts to a second total water use based incentive scheme by promoting the cooperative behavior. We have shown how cooperative management institutions could reduce water overuse, improving incentives for efficient water use, by inducing peer monitoring by cooperative members. We have demonstrated that water overuse is more likely when punishments are weak and cooperatives are large. The basic analysis is then extended to allow first for collusion in monitoring between cooperative members. Our theoretical treatment has shown that the collusive monitoring effort is efficient. Secondly, we have studied a different monitoring structure "Rotating monitoring" (although in practice the mutual monitoring structure is commonly observed) and compared it with the mutual monitoring structure. Finally, we have tried to use some theoretical results to derive some usefully policy recommendations that could help decision makers to implement the right policies to alleviate water stress.

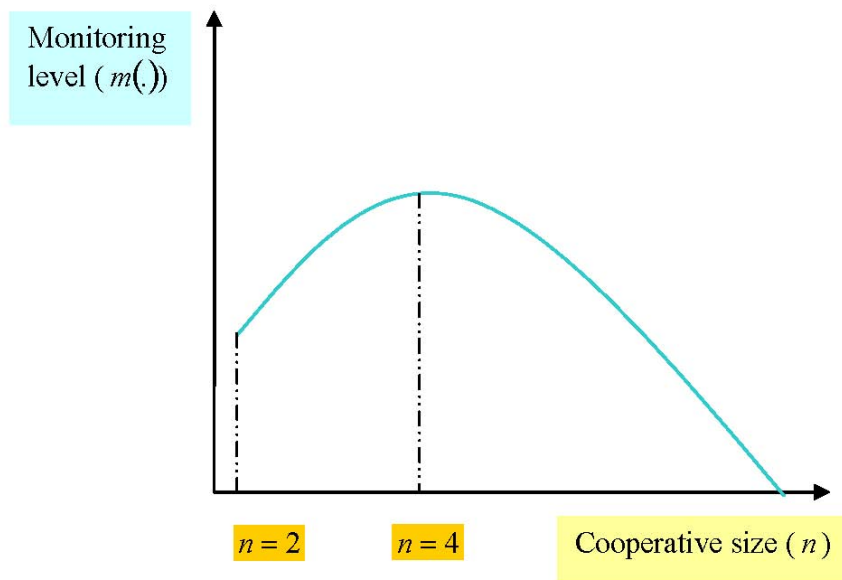
Overall, these results provide strong confirmation of the ability of well designed incentives to reduce the overuse of the resource water. Since the total water use by the group is publicly observable, this theoretical treatment proves the effectiveness of total water use-based incentive schemes compared to individual water use-based incentive scheme. We also have confirmation of the tendency of cooperative institutions to adapt to the level of monitoring costs. Higher monitoring costs have a positive direct effect on the incidence of water overuse, but a negative indirect effect by inducing farmers to reduce cooperative size and increase punishment levels.

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Figure 1:



The first-order conditions above are both necessary and sufficient to identify a global maximum because program (P) is convex (i.e. the objective function is concave and the constraints are linear). This completes the proof of proposition 1.

B. The proof of proposition 2

We solve this game by backward induction. At stage 2 of the game, farmer i optimally chooses the amount of water to use, $q_i^c \equiv q_i^c(m_i, m_j)$ which maximizes his expected payoff, given the levels of monitoring performed by the two cooperative members, m_i and m_j and that farmer j chooses $q_j^c \equiv q_j^c(m_i, m_j)$

$$\max_{q_i} U_i(m_i, q_i). \quad (\text{B1})$$

Where

$$U_i(m_i, q_i) = g(q_i) - cq_i - t\bar{q} - \frac{1}{2}f(1 - \kappa m_i + \kappa m_j)[(q_i - \bar{q}) + (q_j - \bar{q})] - \psi(m_i). \quad (\text{B2})$$

The first-order condition with respect to q_i is given by

$$q_i^c: g'(q_i) - c - \frac{1}{2}f(1 - \kappa m_i + \kappa m_j) = 0, \quad (\text{B3})$$

At stage 1 of the game, farmer i chooses m_i^c (given that farmer j chooses m_j^c) so as to solve

$$\max_{m_i} g(q_i) - cq_i - t\bar{q} - \frac{1}{2}f(1 - \kappa m_i + \kappa m_j)[(q_i - \bar{q}) + (q_j - \bar{q})] - \psi(m_i) \text{ for } i \neq j.$$

whose first-order condition

$$\frac{\partial U_i^c}{\partial m_i} = \frac{1}{2}\kappa f[(q_i - \bar{q}) + (q_j - \bar{q})] - \frac{1}{2}f(1 - \kappa m_i + \kappa m_j) \frac{\partial(q_j - \bar{q})}{\partial m_i} - \psi'(m_i). \quad (\text{B4})$$

Now we differentiate (B4) with respect to m_i , which gives

$$\frac{\partial^2 U_i}{\partial m_i^2} = \frac{1}{2}\kappa f \left[\frac{\partial(q_i - \bar{q})}{\partial m_i} + \frac{\partial(q_j - \bar{q})}{\partial m_i} \right] - \frac{1}{2}f(1 - \kappa m_i + \kappa m_j) \frac{\partial^2(q_j - \bar{q})}{\partial m_i} - \psi''(m_i), \quad (\text{B5})$$

where,

$$\frac{\partial(q_i - \bar{q})}{\partial m_i} = -\frac{\kappa f}{2g''(q_i)}; \quad \frac{\partial(q_j - \bar{q})}{\partial m_i} = \frac{\kappa f}{2g''(q_j)} \text{ and } \frac{\partial^2(q_j - \bar{q})}{\partial m_i} = \frac{(\kappa f)^2}{4} \left(-\frac{g'''(q_j)}{[g''(q_j)]^3} \right) \quad (\text{B6})$$

By replacing equation (B6) into equation (B5), yields

$$\frac{\partial^2 U_i}{\partial m_i^2} = \frac{1}{2}\kappa f \left(-\frac{\kappa f}{2g''(q_i)} + 2\frac{\kappa f}{2g''(q_j)} \right) - \frac{1}{2}f(1 - \kappa m_i + \kappa m_j) \frac{(\kappa f)^2}{4} \left(-\frac{g'''(q_j)}{[g''(q_j)]^3} \right) - \psi''(m_i), \quad (\text{B7})$$

We will focus on the symmetric subgame perfect equilibrium where $m_i^c = m_j^c = m^c$ and $q_i^c = q_j^c = q^c$ which is given by:

The equilibrium amount of water use

$$g'(q^c) = c + \frac{1}{2}f, \quad (\text{B8})$$

The equilibrium monitoring effort m^c is given by

$$m^c = \phi(k_2) \quad (\text{B9})$$

where $k_2 = f \left[(q^c - \bar{q}) - \frac{1}{4} \frac{f}{g''(q^c)} \right]$ and $\phi = (\psi')^{-1}$

and the first-order condition for the level of monitoring m^c which is given by (B5) is also sufficient because the second partial derivative of the farmer's utility function is negative

$$\frac{\partial^2 U_i}{\partial m_i^2}(m^c, q^c) = \left(\frac{(\kappa f)^2}{4g''(q^c)} \right) \left[1 + \frac{f}{2} \left(\frac{g'''(q^c)}{[g''(q^c)]^2} \right) \right] - \psi''(m^c) < 0 \quad (\text{B10})$$

This completes the proof of proposition 2

C. Corollary 2

Consider what happens if the two cooperative members choose their monitoring efforts in order to maximize joint payoffs, $U_i(q_i, m_i) + U_j(q_j, m_j)$

$$U_i(q_i, m_i) + U_j(q_j, m_j) = \left\{ \begin{array}{l} \left[g(q_i) - cq_i - t\bar{q} - \frac{1}{2}f(1 - \kappa m_i + \kappa m_j)(a_i + a_j) - \psi(m_i) \right] \\ + \left[g(q_j) - cq_j - t\bar{q} - \frac{1}{2}f(1 - \kappa m_j + \kappa m_i)(a_i + a_j) - \psi(m_j) \right] \end{array} \right\} \quad (\text{C1})$$

We solve the game by backward induction. We start with the second stage of the game, where farmers choose their water use levels, q_i and q_j so as to maximize their joint payoffs:

$$\max_{(q_i, q_j)} U_i(q_i, m_i) + U_j(q_j, m_j).$$

whose first order conditions with respect to q_i and q_j are respectively given by

$$q_i: g'(q_i) = c + \frac{1}{2}f(1 - \kappa m_i + \kappa m_j), \quad (\text{C2})$$

and

$$q_j: g'(q_j) = c + \frac{1}{2}f(1 - \kappa m_j + \kappa m_i), \quad (\text{C3})$$

At the first stage of the game, cooperative members choose m_i and m_j so as to maximize their joint payoffs:

$$\max_{(m_i, m_j)} U_i(q_i(m_i; m_j), m_i) + U_j(q_j(m_j; m_i), m_j).$$

whose first-order conditions with respect to m_i and m_j are respectively given by

$$\left\{ \begin{array}{l} \left[-\frac{1}{2}f(1 - \kappa m_i + \kappa m_j) \frac{\partial q_j}{\partial m_i} + \frac{1}{2}f(a_i + a_j) - \psi'(m_i) \right] \\ + \left[-\frac{1}{2}f(a_i + a_j) - \frac{1}{2}f(1 - \kappa m_j + \kappa m_i) \frac{\partial q_j}{\partial m_i} \right] \end{array} \right\} = 0$$

(C4)

and

$$\left\{ \begin{array}{l} \left[-\frac{1}{2}f(1 - \kappa m_i + \kappa m_j) \frac{\partial q_j}{\partial m_j} + \frac{1}{2}f(a_i + a_j) \right] \\ + \left[\frac{1}{2}f(a_i + a_j) - \frac{1}{2}f(1 - \kappa m_j + \kappa m_i) \frac{\partial q_j}{\partial m_j} - \psi'(m_j) \right] \end{array} \right\} = 0$$

(C5)

By rearranging the above equations, one gets the followings expressions

$$m_i: \frac{1}{4}f^2\kappa \left[(1 - \kappa m_j + \kappa m_i) \frac{1}{g''(q_i)} - (1 - \kappa m_i + \kappa m_j) \frac{1}{g''(q_j)} \right] = \psi'(m_i)$$

(C6)

and

$$m_j: \frac{1}{4}f^2\kappa \left[(1 - \kappa m_j + \kappa m_i) \frac{1}{g''(q_i)} - (1 - \kappa m_i + \kappa m_j) \frac{1}{g''(q_j)} \right] = \psi'(m_i)$$

(C7)

Equations (C6) and (C7) above imply that

$$\psi'(m_i) = -\psi'(m_j)$$

(C8)

Since $\psi'(m_k)$ for $k = i, j$ is non negative

$$\psi'(m_i) = -\psi'(m_j) \geq 0.$$

(C9)

The above equality (C9) holds only if the two terms are equal to zero, i.e.

$$\psi'(m_i) = -\psi'(m_j) = 0.$$

(C10)

which implies that the coordinated monitoring efforts are

$$m_i = m_j = 0. \quad (\text{C11})$$

This completes the proof of corollary 2.

D. Mutual Monitoring

We solve the game by backward induction. At stage 2 of the game, farmer i chooses the water use level $q_i^c(m_i, m_j, m_k)$ that maximizes his expected payoff, given that his peers choose the equilibrium water use levels, $q_j^c(m_i, m_j, m_k)$ and $q_k^c(m_i, m_j, m_k)$

$$\begin{aligned} \max_{q_i} U(q_i, m_i; m_j, m_k) = & g(q_i) - cq_i - t\bar{q} - \frac{f}{3} S_i^3 [(q_i - \bar{q}) + (q_j - \bar{q}) + (q_k - \bar{q})] \\ & - \psi(2m_i). \end{aligned} \quad (\text{D1})$$

whose first order condition is

$$g'(q_i) = c + \frac{f}{3} S_i^3(m_i, m_j, m_k) \quad (\text{D2})$$

At stage 1 of the game, farmer i chooses the monitoring effort, m_i^c given that his peers choose the equilibrium monitoring efforts m_j^c and m_k^c

$$\begin{aligned} -\frac{f}{3} S_i^3 \left(\frac{\partial q_j}{\partial m_i} + \frac{\partial q_k}{\partial m_i} \right) \\ - \frac{f}{3} \left\{ \left[\begin{aligned} & -2\kappa(1 - \kappa m_i)(1 - \kappa m_j)(1 - \kappa m_k) \\ & + (\kappa m_j + \kappa m_k - \kappa^2 m_j m_k) \left(\frac{1}{2} [-\kappa(1 - \kappa m_k) - \kappa(1 - \kappa m_j)] \right) \right] \right\} \\ & [(q_i - \bar{q}) + (q_j - \bar{q}) + (q_k - \bar{q})] \end{aligned} \right\} \end{aligned} \quad (\text{D3})$$

where,

$$\begin{aligned} \frac{\partial q_j}{\partial m_i} = & \frac{f}{3g''(q_j)} \left\{ \kappa(1 - \kappa m_j)^2 (1 - \kappa m_k) \right. \\ & \left. + (\kappa m_j + \kappa m_k - \kappa^2 m_j m_k) \left(\frac{1}{2} [-\kappa(1 - \kappa m_k) - \kappa(1 - \kappa m_j)] \right) \right\} \end{aligned} \quad (\text{D4})$$

and

$$\begin{aligned} \frac{\partial q_k}{\partial m_i} = \frac{f}{3g''(q_k)} & \left\{ \kappa(1 - \kappa m_j)(1 - \kappa m_k)^2 \right. \\ & + (\kappa - \kappa^2 m_j) \left(1 + \frac{1}{2} [(1 - \kappa m_j)(1 - \kappa m_k) + (1 - \kappa m_i)(1 - \kappa m_j)] \right) \\ & \left. + (\kappa m_j + \kappa m_k - \kappa^2 m_j m_k) \left(\frac{1}{2} [-\kappa(1 - \kappa m_k)] \right) \right\} \end{aligned} \quad (D5)$$

We restrict attention to the symmetric equilibrium where $q_i^c = q_j^c = q_k^c = q_3^c$ and $m_i^c = m_j^c = m_k^c = m_3^{c(M)}$ assuming that the second-order condition for a maximum holds)

$$q_3^c: g'(q) = c + \frac{1}{3}f, \quad (D6)$$

and the equilibrium rotative monitoring effort satisfies the following implicit equation

$$\begin{aligned} m_3^{c(M)}: -\frac{f^2}{9g''(q^c)} & \left[\kappa^4 m^4 + \frac{1}{2} \kappa^4 m^3 - 2\kappa^3 m^3 - \frac{5}{2} \kappa^3 m^2 + 3\kappa^3 m \right] - f(q^c \\ & - \bar{q}) [\kappa^4 m^3 - 3\kappa^3 m^2 + 4\kappa^2 m] \end{aligned} \quad (D7)$$

This completes the proof.

E. Rotative Monitoring

We solve the game by backward induction. At stage 2 of the game, farmer i chooses the water use level $q_i^c(m_i, m_j, m_k)$ that maximizes his expected payoff, given that his peers choose the equilibrium water use levels, $q_j^c(m_i, m_j, m_k)$ and $q_k^c(m_i, m_j, m_k)$.

$$\begin{aligned} \max_{q_i} U(q_i, m_i; m_j, m_k) & = g(q_i) - cq_i - t\bar{q} - \frac{f}{3} S_i^3 [(q_i - \bar{q}) + (q_j - \bar{q}) + (q_k - \bar{q})] \\ & - \psi(m_i). \end{aligned}$$

whose first order condition is

$$g'(q_i) = c + \frac{f}{3} S_i^3(m_i, m_j, m_k) \quad (E1)$$

At stage 1 of the game, farmer i chooses the monitoring effort, m_i^c given that his peers choose the equilibrium monitoring efforts m_j^c and m_k^c

$$-\frac{f}{3}S_i^3 \left(\frac{\partial q_j}{\partial m_i} + \frac{\partial q_k}{\partial m_i} \right) - \frac{f}{3} \left\{ \left[\begin{array}{c} \kappa^3 m_j m_k - \kappa(1 - \kappa m_j)(1 - \kappa m_k) \\ + \frac{3}{2} [\kappa^2 m_k (1 - \kappa m_j) - \kappa^3 m_j m_k] - 3\kappa^2 m_k (1 - \kappa m_j) \\ [(q_i - \bar{q}) + (q_j - \bar{q}) + (q_k - \bar{q})] \end{array} \right] \right\} \quad (\text{E2})$$

where,

$$\frac{\partial q_j}{\partial m_i} = \frac{f}{3g''(q_j)} \left\{ \begin{array}{c} \kappa^3 m_j m_k - \kappa(1 - \kappa m_j)(1 - \kappa m_k) \\ + \frac{3}{2} [\kappa^2 m_k (1 - \kappa m_j) + \kappa^2 m_j (1 - \kappa m_k)] + 3\kappa \end{array} \right\} \quad (\text{E3})$$

and

$$\frac{\partial q_k}{\partial m_i} = \frac{f}{3g''(q_k)} \left\{ \begin{array}{c} \kappa^3 m_j m_k - \kappa(1 - \kappa m_j)(1 - \kappa m_k) + \frac{3}{2} [\kappa^2 m_j m_k + \kappa^2 m_j (1 - \kappa m_k)] \\ - 3\kappa^2 m_j \end{array} \right\} \quad (\text{E4})$$

We characterize the symmetric subgame perfect equilibrium where $q_i^c = q_j^c = q_k^c = q_3^c$ and $m_i^c = m_j^c = m_k^c = m_3^c$ (assuming that the second-order condition for a maximum holds)

$$q_3^c: g'(q) = c + \frac{1}{3}f, \quad (\text{E5})$$

and the equilibrium rotative monitoring effort satisfies the following implicit equation

$$m^{c(R)}: -\left(\frac{1}{2}\kappa^2 m + \kappa\right) \left[-\frac{f^2}{9g''(q^c)} + f(q^c - \bar{q}) \right] = \psi'(m).$$

This completes the proof.