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ASSET ALLOCATION WITH LIQUIDITY-ADJUSTED MARKET RISK MODELING: EMPIRICAL RELEVANCE TO EMERGING GCC FINANCIAL MARKETS

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Abstract

The aim of this article is to bridge the gap in equity trading risk management literatures and particularly from the perspective of emerging and illiquid markets, such as in the context of the Gulf Cooperation Council (GCC)'s six financial markets. To the authors' best knowledge, this is the first research paper that addresses the issue of equity trading risk management in the GCC countries with direct applications to their six stock markets. In this paper, the authors demonstrate a practical approach for measurement, management and control of market and liquidity risk exposures for financial trading portfolios that contain several illiquid equity securities. This approach is based on the renowned concept of Liquidity-Adjusted Value at Risk (L-VaR) along with the development of an optimization software tool utilizing matrix-algebra technique under the notion of different correlation factors and liquidation horizons. The comprehensive trading risk model can simultaneously handle L-VaR analysis under normal and severe market conditions besides it takes into account the effects of illiquidity of all traded equity securities. In order to illustrate the proper use of L-VaR and stress-testing methods, real-world examples and feasible reports of equity trading risk management are presented for the six GCC equity financial markets by implementing a daily database of indices' returns for the period 2004-2008. To this end, several financial modeling studies are achieved with the objective of creating a realistic framework of equity trading risk measurement and control reports in addition to the instigation of a practical iterative optimization technique for the calculation of maximum authorized L-VaR limits subject to real-world optimum operational constraints.

ملخص

يهدف هذا البحث إلى سد الفجوة في أدبيات إدارة مخاطر تداول الأسهم وبصفةٍ خاصةٍ من منظور الأسواق الناشئة والتي لا تعتمد على السيولة؛ على سبيل المثال في سياق الأسواق المالية الستة لمجموعة دول مجلس التعاون الخليجي. وتعد هذه هي ورقة البحث الأولى التي تتناول بالدراسة مبحث إدارة مخاطر تداول الأسهم في دول مجلس التعاون الخليجي وما لذلك من تطبيقاتٍ مباشرةٍ على أسواق المال الست في هذا التكتل.

ويتبنى مؤلفو هذه الورقة أسلوباً عملياً لقياس وإدارة والتحكم في السوق وكذلك تعرض محافظ التداول المالية لمخاطر السيولة؛ تلك المحافظ التي تحتوي على العديد من أوراق حقوق الملكية غير السائلة.

ويقوم هذا المنهج على المفهوم الشهير "القيمة المعدلة للسيولة المعرضة للخطر" جنباً إلى جنبٍ مع تطوير أداةٍ برمجيةٍ ذات فعالية باستخدام طريقة الجبر والمصفوفات في إطار مفهوم ترابط العوامل المختلفة وآفاق التصفية.

وبإمكان النموذج الشامل لمخاطر التداول أن يعالج تحليل "القيمة المعدلة للسيولة المعرضة للخطر" في ظروف السوق العادية والصعبة آخذاً في الاعتبار في الوقت ذاته تأثيرات اللاسيولة على جميع أنواع أوراق الملكية المتداولة.

ولبيان الاستخدام الأمثل للقيمة المعدلة للسيولة المعرضة للخطر وطرق اختبارات الإجهاد فقد تم تقديم أمثلةٍ وتقارير جدوى واقعيةٍ لإدارة المخاطر في تداول الأسهم من واقع الأسواق المالية الست في تجمع دول مجلس التعاون الخليجي وذلك من خلال إنشاء قاعدة بيانات بشكلٍ يومي من أرباح المؤشرات خلال الفترة من العام 2004 حتى العام 2008. وسعياً وراء هذه الغاية فقد تم القيام بعديدٍ من دراسات النمذجة بهدف خلق إطارٍ واقعيٍ لقياس مخاطر تداول الأسهم وتقارير الرقابة بالإضافة إلى المضي قدماً نحو تبني تقنيةٍ تكراريةٍ عمليةٍ في حساب الحدود القصوى من القيمة المعدلة للسيولة المعرضة للخط

1. Introduction and Overview

In the last two decades, financial institutions in emerging markets have greatly increased their holdings of trading assets, such as bonds, equities, interest rate and equity derivatives, foreign exchange and commodity positions. Their intention in this has been to earn trading profits and to hedge exposures elsewhere in their trading portfolios. Nevertheless, the lack of adequate market risk measurement, management, and control tools are some of the contributing factors that have led to major financial losses among national/multinational corporations in emerging economies.

To quantify the risks involved in their trading operations, major financial institutions are increasingly exploiting Value at Risk (VaR) models. Since financial institutions differ in their individual characteristics, tailor-made internal risk models are more appropriate. Moreover, the increase in the relative importance of trading risk in financial institutions' portfolios has obliged regulators to reconsider the system of capital requirements as outlined in the previous Basel Capital Accords. Fortunately, and in accordance with the latest Basel Capital Accord, financial institutions are permitted to develop their own internal risk models for the purposes of providing for adequate risk measures. Furthermore, internal risk models can be used to determine the capital that banks must hold to endorse their trading of securities. The benefit of such an approach is that it takes into account the relationship between various asset types and can accurately assess the overall risk for a whole combination of trading assets.

For establishing adequate internal models of risk management, the new Basel Accord (Basel II) has motivated several emerging countries to be part of the agreement at different implementation levels. This is aggravated by the fact that emerging markets financial institutions face a substantial competitive disadvantage if they are enforced to continue using the standardized approach. As such, several emerging markets in the Asian and Latin American continents would like to be Basel II-compliant, and hence are already in advanced steps to implement, by the end of the year 2008, modified versions of the Basel agreement with its suggested internal models. Basel II's overall intention is to endorse adequate capitalization of banks, and encourage improvements in risk measurement, management and control, thereby strengthening stability in the whole financial system. Basel II does so by implementing three complementary pillars: one concerning capital adequacy methodology and calculation, another on supervisory review, and a third setting disclosure terms to enable market discipline.

A number of Arab countries are voluntarily joining the implementation of modified versions of Basel II. In fact, the GCC's financial markets, in general, are in progressive stages of implementing advanced risk management regulations and techniques. Moreover in recent years, outstanding progress has been done in cultivating the culture of risk management among local financial entities and regulatory institutions. In the Middle East, the majority of banking assets is expected to be covered by Basel II regulations during 2007-2009. Generally speaking, capital ratios are fairly strong in the GCC, though they have fallen lately as banks have expanded their products and operations. Within the GCC, there have been negotiations for common application of Basel II rules, though with different timeframes. This is due to the fact that some GCC countries are more diverse, for instance, in terms of the presence of foreign banks than others.

The financial industry in GCC countries is generally sound, and the six countries continue to develop their financial system to attract more foreign portfolio investors, and to expand the opening of their financial system to the exterior world. Consequently, several local financial institutions are in a consolidation route, while some others have already followed a process of convergence for their financial operations and have already started the procedure of modernizing their internal risk management capabilities. By the standards of emerging market countries the quality of banking supervision in the six GCC states is well above average. Despite the latest progress in the GCC financial markets to become Basel-compliant countries, recently it has been deemed necessary (by local regulatory authorities) to adopt proper internal risk models, rules and procedures that financial entities, regulators and policymakers should consider in setting-up their daily trading risk management objectives.

Despite the increasing importance of trading risk management, published research in this specific risk management area is slow to emerge, specifically from the perspective of market practitioners. In particular, the main aim of this paper is to fill a gap in the equity risk management literature (especially from the perspectives of emerging and illiquid markets) and to bridge the disparity between the academic and professional finance communities. This paper imparts equity risk management methodology and modeling techniques (which are drawn from a practitioner's viewpoint) that can be applied to emerging markets' equity portfolio investments and also to the day-to-day equity trading activities. In this work, key equity market risk management methods and optimization techniques that financial entities, regulators and policymakers should consider in setting-up their daily equity trading risk management objectives are examined and adapted to the specific needs of emerging and illiquid markets, such as in the context of the six GCC stock markets. The suggested quantitative methods and modeling techniques can be implemented in almost all emerging economies, if they are adapted to correspond to each market's initial level of sophistication. To this end, this study investigates the application of modern financial theory and financial risk management tools and techniques to the case of emerging markets' trading portfolios that contain vast amount of illiquid equity cash securities. It also provides an insight on how to measure and report daily trading risk exposure of both long and short trading positions, within its authorized risk limits constraints, in innovative and proactive ways to senior management in financial entities.

More specifically, the intent of this paper is to propose a realistic approach for the inclusion of liquidation trading risk in standard VaR analysis. The key methodological contribution is a different and less conservative liquidity scaling factor than the conventional root-*t* multiplier. The proposed add-on is a function of a predetermined liquidity threshold defined as the maximum position which can be unwound without disturbing market prices during one trading day. In addition, the reengineered model is quite simple to implement even by very large financial institutions with multiple assets and risk factors. In fact, the essence of the model relies on the assumption of a stochastic stationary process and some rules of thumb, which can be of crucial value for more accurate market risk quantification under illiquid market conditions. In this paper, we attempt to integrate and estimate the impact of liquidity trading risk into VaR models by explicitly incorporating the impact of the time-volatility dimension of liquidity risk instead of the movements in the bid-ask spread. The approach to assessing liquidity-adjusted VaR for distinctive equity portfolio has been illustrated with the help of a modified closed-form parametric model. We then demonstrate, by applying the liquidity risk measures to the GCC's financial markets, to what extent the quantified liquidity trading risk effects can impact traditional measurement of market risk under different correlation assumptions: empirical, zero and unity.

To this end, the parameters required for the construction of appropriate and simplified Liquidity-Adjusted Value at Risk (L-VaR) and stress-testing methods are reviewed from previous work and adapted to the specific applications of these methods to emerging markets. The theoretical analytical models that are applied herein are based on matrix-algebra approach. The latter approach can simplify the iterative-optimization programming process and permits easy incorporation of short-selling of assets into the equity trading process. Moreover, a simplified proactive approach for the incorporation of illiquid assets, in daily trading risk management practices, is defined and is appropriately integrated into L-VaR and stress-testing models. As such, trading risk management models developed in this work are applied to the six GCC stock markets. Firstly, several tests of abnormal (asymmetric) distributions of returns are performed. To this end, various tests of skewness, kurtosis and Jarque-Bera (JB) statistics are implemented. This is followed by a study of daily volatilities (under normal and severe market conditions) along with the calculations of sensitivity factors of the sample indices against a benchmark index. Furthermore, several case studies are carried out with the objectives of calculating L-VaR numbers under the notion of various market scenarios and correlation factors. The different market portfolio scenarios are performed by implementing distinct asset allocation ratios and unwinding periods, and by taking into account the possibilities of short-selling in daily equity trading operations. Our case analysis and studies demonstrate L-VaR numbers under the normal market condition along with a severe crisis condition (stressed or abnormal market situations) with different liquidation horizons and correlation factors.

2. A Literature Review on Market Risk Models

2.1. Literatures Related to Value at Risk (VaR) Common Approaches

Risk management has become of paramount importance in the financial industry and a major endeavor by academics, practitioners, and regulators. A cornerstone of recent interest is a class of models called Value at Risk (VaR) techniques. The concepts of VaR and other advanced risk management techniques are not new and are based—with some modifications—on modern portfolio theory. Even though VaR is one of many—both quantitative and qualitative—factors that should be integrated into a cohesive risk management approach, it is remarkably a vital one. In fact, VaR is not the maximum loss that will occur, but rather a loss level threshold that will be pierced some percentage of the time. The actual loss that occurs could be much higher than the VaR estimates. As such, VaR should be used in conjunction with other risk measures such as stress-testing, scenario analysis, and other asset/business specific risk measures. The most common VaR models estimate variance/covariance matrices of asset returns using historical time series, under the assumption that asset returns are normally distributed, and that portfolio risk is a function of the risk of each asset and the correlation factors among the returns of all trading assets within the portfolio. The VaR is then calculated from the standard deviation of the portfolio, given the appropriate investment/liquidation horizon, and the specified confidence interval.

Despite many criticisms of the limitations of the VaR method, it has proven to be a very useful measure of market risk, and is widely used in financial and non-financial markets. The RiskMetrics™ system (1994), developed and popularized by J. P. Morgan, has provided a tremendous impetus to the growth in the use of VaR concept and other modern risk management techniques and procedures. Since then the VaR concept has become well known, and scores of specific applications are adapted to credit risk management and mutual funds investments. The general recognition and use of large scale VaR models has initiated a considerable literature including statistical descriptions of VaR and assessments of different modeling techniques. For a comprehensive survey, and the different VaR analysis and techniques, one can refer to Jorion (2001). For the most part, VaR analyses in the public domain have been limited to comparing different modeling approaches and implementation procedures using illustrative portfolios [see for instance, Hendricks (1996), Marshall and Siegel (1997), Pritsker (1997)].

In their paper Berkowitz and O'Brien (2001) question how accurate VaR models are in commercial banks. Due to the fact that trading accounts of large commercial banks have considerably grown and become increasingly diverse and complex, the authors presented statistics on trading revenues from such activities, and on the associated VaR forecasts internally estimated by banks. Several other authors have attempted to tackle the issues of extreme events and fat tails phenomena in the distribution of returns. While most of their approaches and techniques are good exercises for academic purposes they do lack evidence of real-world applications with actual market portfolios. For instance, a more rigorous mathematical treatment of VaR analysis with dynamic copula models and extreme value theory has received considerable treatment from Embrechts, et al. (2003).

In their article Angelidis and Degiannakis (2005), they enumerate the accuracy of parametric, nonparametric and semi-parametric methods in predicting the one-day-ahead VaR in three types of markets (namely stock exchanges, commodities and foreign exchange rates) and for both long and short trading positions. In another study, Bredin and Hyde (2004) measure and evaluate the performance of a number of VaR methods that have proven popular through using an equally weighted portfolio based on the foreign exchange exposure of a small open economy (Ireland) among its six major trading partners. Accordingly, their findings suggest that the Orthogonal GARCH model is the most accurate methodology while the exponential weighted moving average (EWMA) specification is the more conservative approach.

According to Culp et al. (1998), VaR can be adapted for use in asset management and for the estimation of market risk in the long-term. In their study, they explore the application of VaR to asset management and with particular attention on the importance of VaR for multi-currency asset managers. Garcia et al. (2007) tackle a specific issue within the VaR which is the sub-additivity property required for the VaR to be a coherent measure of risk. The authors argue that, in the context of decentralized portfolio management, central management possesses only a fraction of information that belongs to each specialist (trader). In such a context, a distribution appears always thicker to the central unit than to the specialist. Therefore, because of a lack of information, VaR may appear fallaciously non sub-additive to the central management unit. Despite evidence to the contrary, the authors show that decentralized portfolio management with a VaR allocation to each specialist will work, and furthermore VaR remains sub-additive in many situations of practical interest.

Finally, in his research paper Al Janabi (2005) establishes a practical framework for the measurement, management and control of trading risk. The effects of illiquid assets, which are dominant characteristics of emerging markets, are also incorporated in his models. The established models and the general framework of risk calculations are mainly based on matrix-algebra techniques. In a first attempt, Al Janabi (2007b) performs a rigorous analytical risk management study on the Mexican Stock Market (BMV). The parameters of a practical framework for the management of market risk are illustrated and a case study is carried out on 11 well-known Mexican stocks besides a number of sector indices and main market indicators. Moreover, in one of his most recent research papers, Al Janabi (2007a), the robust market risk models are applied to large foreign exchange trading portfolios that consist of long and short-selling positions of multi-currencies of numerous developed and emerging economies.

2.2. Literature Related to Liquidity-Adjusted Value at Risk (L-VaR) Modeling

Methods for measuring market (or trading) risk have been well developed and standardized in the academic as well as the banking world. Liquidity trading risk, on the other hand, has received less attention, perhaps because it is less significant in developed countries where most of the market risk methodologies originated. In all but the most simple of circumstances, comprehensive metrics of liquidity trading risk management do not exist. Nonetheless, the combination of the recent rapid expansion of emerging markets' trading activities and the recurring turbulence in those markets has propelled liquidity trading risk to the forefront of market risk management research and development.

Indeed, liquidity trading risk management is of vital importance to entities in the financial services sector. Most collapses of financial entities have occurred in large part due to insufficient liquidity resulting from undesirable events. Accordingly, the need for better measurement and management tools of liquidity trading risk are on the rise in the wide-reaching financial markets. This is with the principal objective of setting a comprehensive set of liquidity and funding policies that are intended to maintain significant flexibility to address market liquidity events and to enable core trading activities to continue to generate revenue even under adverse circumstances.

As such, liquidity trading risk is an all-embracing apprehension for anyone holding a portfolio of any type of trading asset, and liquidity crises proves to be imperative in the failure of many financial entities. More specifically, liquidity trading risk arises from situations in which a party interested in trading an asset cannot do so because no one in the market wants to trade that asset. Liquidity risk becomes for the most part important to financial market participant who are about to hold or currently hold an asset, since it affects their aptitude to trade or unwind the trading position. Insolvencies often occur because financial entities cannot get out or unwind their holdings effectively and hence the liquidation value of assets may differ significantly from their current mark-to-market values.

The conventional VaR approach to computing market (or trading) risk of a portfolio does not explicitly consider liquidity risk. Typical VaR models assess the worst change in mark-to-market portfolio value over a given time horizon but do not account for the actual trading risk of liquidation. Customary fine-tunings are made on an ad hoc basis. At most, the holding period (or liquidation horizon) over which the VaR number is calculated is adjusted to ensure the inclusion of liquidity risk. As a result, liquidity trading risk can be imprecisely factored into VaR assessments by assuring that the liquidation horizon is at a minimum larger than an orderly liquidation interval. Moreover, the same liquidation horizon is employed to all trading asset classes, albeit some assets may be more liquid than others. Neglecting liquidity risk can lead to an underestimation of the overall market risk and a misapplication of capital cushion for the safety and soundness of financial institutions. In emerging financial markets, which are relatively well thought-out as illiquid, ignoring the liquidity risk can result in significant underestimation of the VaR estimate, and especially so under severe market conditions.

As such, the combination of the recent rapid expansion of emerging markets trading activities and the recurring turbulence in those markets has propelled liquidity trading risk measurement and management to the forefront of market risk management research. Within the VaR framework, Jarrow and Subramanian (1997) provide a market impact model of liquidity by considering the optimal liquidation of an investment portfolio over a fixed horizon. They derive the optimal execution strategy by determining the sales schedule that will maximize the expected total sales values, assuming that the period until liquidation is given as an exogenous factor. The correction to the lognormal VaR they derive depends on the mean and standard deviation of both an execution lag function and a liquidation discount. Although the model is simple and intuitively appealing, it suffers from practical difficulties for its implementation. It requires the estimation of additional parameters such as the mean and the standard deviation of the discount factor and the period of execution—for which data are not readily available, none of which may be easy to estimate and may require subjective estimates such as a trader's intuition.

Bangia et al. (1999) approach the liquidity risk from another angle and provide a model of VaR adjusted for what they call exogenous liquidity—defined as common to all market players and unaffected by the actions of any one participant. It comprises such execution costs as order processing costs and adverse selection costs resulting in a given bid-ask spread faced by investors in the market. On the contrary, endogenous liquidity is specific to one's position in the market and depends on one's actions and varies across market participants. It is mainly driven by the size of the position: the larger the size, the greater the endogenous illiquidity. They propose splitting the uncertainty in market value of an asset into two parts: a pure market risk component arises from asset returns and uncertainty due to liquidity risk.

Their model consists of measuring exogenous liquidity risk, computed using the distribution of observed bid-ask spreads and then integrating it into a standard VaR framework.

Le Saout (2002) applies the model developed by Bangia et al. (1999) to the French stock market. In an attempt to consider the effect of liquidating large positions, Le Saout (2002) incorporates the weighted average spread into the Liquidity-Adjusted Value-at-Risk measure (L-VaR). The author's results indicate that exogenous liquidity risk, for illiquid stocks, can represent more than a half of the total market risk. Furthermore, the author extends the model to incorporate endogenous liquidity risk and shows that it represents an important component of the overall liquidity risk. Roy (2004) relates the model provided by Bangia et al. (1999) to the Indian debt market. First, the author presents a comprehensive survey of liquidity adjusted VaR models and then adopts a modified version of the exogenous liquidity approach suggested by Bangia et al. (1999). In that paper, a measure of L-VaR, based on bid-ask spread, is presented and the liquidity risk found to be an important component of the aggregate risk absorbed by financial institutions.

In a recent study, Angelidis and Benos (2006) apply L-VaR measures to the Athens Stock Exchange by incorporating bid-ask variation and the price effect of position liquidation. Their study focuses on the use of high frequency transaction level data of stocks besides sorting out each stock according to their average transaction prices and capitalization. Furthermore, the results indicate that adverse selection increases with trade size while the cost component of the bid-ask spread decreases. Based on these findings, endogenous and exogenous liquidity risks are linked to spread components. For high-priced, high-capitalization stocks, it is found that a VaR correction for illiquidity is not necessary since liquidity risk represents only 3.4% of total market risk. On the other hand, for low capitalization stocks, the percentage of risk due to illiquidity reaches as much as 11% of total market risk and hence it should not be neglected in the overall assessment of L-VaR.

Almgren and Chriss (1999) present a concrete framework for deriving the optimal execution strategy using a mean-variance approach, and show a specific calculation method. Their approach has a high potential for practical application. They assume that price changes are caused by three factors: drift, volatility, and market impact. Their analysis leads to insights into optimal portfolio trading, relating risk aversion to optimal trading strategy, and to several practical implications including the definition of L-VaR. Unlike Almgren and Chriss (1999), Hisata and Yamai (2000) turn the sales period into an endogenous variable. Their model incorporates the mechanism of the market impact caused by the investor's own dealings through adjusting VaR according to the level of market liquidity and the scale of the investor's position.

Berkowitz (2000) argues that unless the likely loss arising from liquidity risk is quantified, the models of VaR would lack the power to explicate the embedded risk. In practice, operational definitions vary from volume-related measures to bid-ask spreads and to the elasticity of demand. The author asserts that elasticity based measures are of most relevance since they incorporate the impact of the seller actions on prices. Moreover, under certain conditions the additional variance arising from seller impact can easily be quantified given observations on portfolio prices and net flows; and that it is possible to estimate the entire distribution of portfolio risk through standard numerical methods.

Shamroukh (2000) contends that scaling the holding period to account for orderly liquidation can only be justified if we allow the portfolio to be liquidated throughout the holding period. The author extends the *RiskMetrics*^{TM} (1994) approach by explicitly modeling the liquidation of the portfolio over time and by showing that L-VaR can be easily obtained by an appropriate scaling of the variance-covariance matrix. Furthermore, market liquidity risk can be modeled by expressing the liquidation price as a function of trade sizes, thus imposing a penalty on instantaneous unwinding of large position. Following this approach, L-VaR can be viewed as a solution to a minimization problem arising from the trade-off between higher variance associated with slow liquidation and higher endogenous liquidity risk associated with fast liquidation. As a result, the holding period is an output of this model—that is the solution to the minimization problem. In another relevant study, Dowd et al. (2004) tackle the problem of estimating VaR for long-term horizon. In their paper they offer a different, however a rather straightforward approach that avoids the inherited problems associated with the square-root of time rule, as well as those associated with attempting to extrapolate day-to-day volatility forecasts over long horizons.

Lately, in his research paper Al Janabi (2008) establishes a practical framework for the measurement, management and control of trading risk. The effects of illiquid assets, that are dominant characteristics of emerging markets, are also incorporated in the risk models. This literature provides real-world risk management techniques and strategies (drawn from a practitioner viewpoint) that can be applied to equity trading portfolios in emerging markets. The intent is to propose a simple approach for the inclusion of liquidation trading risk in standard VaR analysis and to capture the liquidity risk arising because of illiquid trading positions, by obtaining a Liquidity-Adjusted Value at Risk (L-VaR) estimate. The key methodological contribution is a different and less conservative liquidity scaling factor than the conventional root-*t* multiplier. The proposed add-on is a function of a predetermined liquidity threshold defined as the maximum position which can be unwound without disturbing market prices during one trading day. In addition, the re-engineered model is quite simple to implement even by very large financial institutions with multiple assets and risk factors.

3. Motivation and Explicit Objective of Present Work

In spite of the increasing importance of the GCC's financial markets, there is a limited amount of published research in this respect and particularly within the liquidity trading, risk management context. Moreover, to the best of my knowledge, no work has been published yet in any international literature on liquidity risk management that takes into account the GCC countries as a case study. This study makes the following contributions to the literature in this specific risk management field. Firstly, it represent one of the limited number of practitioners papers that empirically examines equity trading risk management using the actual financial data of the six GCC stock markets. Secondly, the paper in essence proposes a variation on the root-*t* rule of augmenting standard one-day VaR by suggesting a less conservative liquidity scaling factor for including liquidation risk in standard VaR analysis. Thirdly, unlike most empirical studies in this field, this study employs a comprehensive and real-world trading risk management model that considers risk analysis under normal, severe (crisis) and illiquid market conditions. The principal advantage of employing such a model is the ability to capture a full picture of possible loss scenarios of actual trading portfolios under different correlation assumptions — empirical, zero and unity.

This paper aims to capture the liquidity risk arising due to illiquid trading positions and obtain a liquidity-adjusted VaR estimate. In contrast to all existing published literature pertaining to the application of L-VaR method to emerging markets, this paper proposes a new model for assessing a closed-form parametric L-VaR with explicit treatment of liquidity trading risk. The key methodological contribution of this work is to extend L-VaR calculation to allow for a steady liquidation of the portfolio over the holding period and by showing that liquidity risk can be straightforwardly and intuitively integrated into the proposed L-VaR framework. Rather than modeling liquidity trading risk as such, the central focus of this work is to overhaul a wide-ranging and adaptable framework for handling liquidity risk in the overall assessment of trading risk. Its essence relies on the assumption of a stochastic stationary process and some rules of thumb, which can be of crucial value for more accurate market risk assessment during market stress periods when liquidity dries up. The liquidity framework presented in this paper does not incorporate all the aspects of liquidity trading risk. However, it is effective as a tool for evaluating trading risk when the impact of illiquidity of specified financial products is significant.

Furthermore and in contradiction to all accessible published literature relevant to putting into effect L-VaR models to emerging markets, in this work, a genuine model for the measurement of illiquidity of both short and long trading and/or investment positions is incorporated into conventional VaR framework. Contrary to other commonly used liquidity models, the liquidity model applied in this work is more appropriate for real-world trading and investment practices since it considers selling small fractions of long/short trading/investment securities on a daily basis. Moreover, it can be applied to both developed and emerging economies and for both trading and investment financial positions. Although the risk measurement method that is adopted in this work is based mainly on the variance/covariance approach (that assume normal distribution of returns), for emerging and illiquid markets it is possible to correct for the assumption of normality by including stress-testing (under severe market conditions) along with the aggregation of a realistic risk factor that takes into account illiquid securities (Al Janabi, 2005).

For equity trading portfolios, the risk measurement models and procedures are based on the renowned concept of L-VaR as well as on the innovation of an optimization software tool utilizing matrix-algebra techniques. Matrix-algebra method is a useful tactic to avoid mathematical complexity, as more and more securities are added to the portfolio of assets. In addition, it can simplify the iterative-optimization programming process and permits easy incorporation of short (sell) and long (buy) positions in risk management process. The latter effect on trading positions can facilitate the risk management for large equity trading portfolios and aid in setting-up of optimum structure of L-VaR limits.

Market risk management models, which are implemented in this work, are applied to the six GCC stock markets. Daily database, for the period 2004-2008, of stock market indices (9 indices in total) are all gathered, filtered and processed in such a manner so that to create meaningful quantitative analysis and conclusion of market and liquidity risk measurement. Several case studies are carried out with the objectives of calculating L-VaR numbers under various possible scenarios in addition to the inception of a practical framework for the establishment of optimum L-VaR limits setting (or risk budgeting). The different scenarios are performed, first with distinct asset allocation percentages, second by studying the effects of liquidity of trading assets (unwinding period of assets), and finally by taking into account the possibilities of short-selling in daily trading operations. To this end, several case analysis studies are carried out with the objectives of assessing L-VaR under diverse illiquid market conditions. L-VaR estimates have been obtained for various equity trading portfolios in the GCC stock markets. The liquidity-adjusted VaR has been obtained through a modified closed-form parametric L-VaR approach. We then use the results to draw conclusions about the relative liquidity of the different stock markets and the importance of liquidity risk in L-VaR estimate. Furthermore, several tests of abnormal (asymmetric) distributions of returns are performed. To this end, various tests of skewness, kurtosis and Jarque-Bera statistics are implemented on the various stock markets indices. This is followed by a study of daily and annual volatilities along with calculations of betas (sensitivity factors) of the sample stock market indices against a main market indicator, namely the Shuaa Arab index.

The remainder of the paper is organized as follows. The following section lays down the quantitative infrastructure of L-VaR, and its limitations, and a model that incorporates the effects of illiquid assets in daily market and liquidity risk management. First, we show that L-VaR can be derived for a single-asset portfolio assuming uniform liquidation over the holding period. We then derive a general and broad model that incorporates the effects of multiple illiquid assets in daily market risk management by simply scaling the multi-assets' L-VaR matrix. Finally, we demonstrate, by applying the L-VaR measures to the GCC stock markets, to what extent the quantified liquidity effects can affect conventional assessment of trading risk. The results of empirical tests are drawn in the final section along with conclusions and recommendations. A full set of equity trading risk management analysis reports and optimum L-VaR trading limits is included within the appendix.

4. A General Parametric Methodology for the Assessment of Liquidity-Adjusted Value at Risk (l-var)

4.1 Theoretical Foundation of L-VaR Models for Equity Price Risk Management

In essence, VaR is intended to measure the largest amount of money a position or trading portfolio could lose, with a given degree of confidence, over a given time horizon and under normal market conditions. Assuming the return of a financial product follows a normal distribution, linear pay-off profile and a direct relationship between the underlying product and the income, the VaR is to measure the standard deviation of the trading income, which results from the volatility of the different markets, for a certain confidence level. This definition gives latitude in choosing both confidence level and time horizon. In practice, however, many financial and non-financial entities have chosen a confidence interval of 95% (or 97.5% if we only look at the loss side [one-tailed]) for their overall portfolio and a one-day time horizon to calculate VaR. This means that once every 40 trading days a loss larger than indicated is expected to occur. Some entities use a 99% (one-tailed) confidence interval, which would theoretically lead to larger losses once every 100 trading days. Due to fat tails of the probability distribution, such a loss will occur more often and can cause problems in calculating VaR at higher confidence levels. Some entities feel that the usage of a 99% confidence interval would place too much trust on the statistical model and, hence, some confidence level should be assigned to the “art–side” of the risk measurement process. Although the method relies on many assumptions and has its drawbacks, it has gained wide acceptance for the quantification and aggregation of trading risks. As a result of the generalization of this method, economic capital allocations for trading and active investment activities tend to be calculated and adjusted with the VaR method.

To calculate VaR using the variance/covariance (also is known as the parametric, analytical and delta-neutral) method, the volatility of each risk factor is extracted from a pre-defined historical observation period. The potential effect of each component of the portfolio on the overall portfolio value is then worked out. These effects are then aggregated across the whole portfolio using the correlations between the risk factors (which are again extracted from the historical observation period) to give the overall VaR value of the portfolio with a given confidence level. A simplified calculation process of the estimation of VaR risk factors (using variance/covariance method) for a single and multiple assets’ positions is illustrated (Al Janabi, 2005 and 2007a) as follows:

From elementary statistics it is well known that for a normal distribution, 68% of the observations will lie within 1σ (standard deviation) from the expected value, 95% within 2σ and 99% within 3σ from the expected value, thus the VaR of a single asset in monetary terms is:

$$\text{VaR}_i = \alpha * \text{Mark-to-Market Value of Equity}_i * \sigma_i \qquad (1)$$

where α is the confidence level (or in other words, the standard normal variant at confidence level α) and σ_i is the standard deviation (volatility) of the return of the equity security that constitutes the single position. The mark-to-market value of equity indicates the amount of investment in equity *i*. Indeed, equation 1 includes some simplifying assumptions, yet it is routinely used by researchers and practitioners in financial markets for the estimation of VaR for a single trading position.

Trading risk in the presence of multiple risk factors is determined by the combined effect of individual risks. The extent of the total risk is determined not only by the magnitudes of the individual risks but also by their correlations. Portfolio effects are crucial in risk management not only for large diversified portfolios but also for individual equities that depends on several risk factors. For multiple equity assets or a portfolio of equity assets**,** VaR is a function of each individual equity's risk and the correlation factor between the returns on the individual equities, and as follows:

$$VaR_P = \sqrt{\left| \, VaR \, \right|^T * \left| \, \rho \, \right| * \left| \, VaR \, \right|} \tag{2}$$

This formula is a general one for the calculation of VaR for any portfolio regardless of the number of equities. It should be noted that this formula is presented in terms of matrix-algebra — a useful form to avoid mathematical complexity, as more and more securities are added. This approach can simplify the programming process and permits easy incorporation of short-selling positions in market risk management process. This means, in order to calculate the VaR (of a portfolio of any number of equities), one needs first to create a matrix $\left| \, VaR \, \right|$ of individual VaR equity positions — explicitly *n* rows and one column (*n*1*) matrix — a transpose matrix $\left| \, VaR \, \right|^T$ of individual VaR equity positions — an (*1*n*) matrix, and hence the superscript "*T*" indicates transpose of the matrix — and finally a matrix $\left| \, \rho \, \right|$ of all correlation factors between all equities (ρ) — an (*n*n*) matrix. Consequently, as one multiplies the three matrices and then takes the square root of the result, he ends up with the VaR_P of any portfolio with any *n*-number of equities. This simple number summarizes the portfolio's exposure to market risk. Investors and senior managers can then decide whether they feel easy with this level of risk. If the answer is no, then the process that led to the estimation of VaR can be used to decide where to reduce redundant risk. For instance, the riskiest equities can be sold, or one can use derivative securities such as futures and options contracts on equities to hedge the undesirable risk.

VaR method is only one approach of measuring market risk and is mainly concerned with maximum expected losses under normal market conditions. It is not an absolute measure, as the actual amount of loss may be greater than the given VaR amounts under severe circumstances. In extreme situations, VaR models do not function very well. As a result, for prudent risk management and as an extra management tool, firms should augment VaR analysis with stress-testing and scenario procedures. From a risk management perspective, however, it is desirable to have an estimate for what potential losses could be under severely adverse conditions where statistical tools do not apply. As such, Stress-testing estimates the impact of unusual and severe events on the entity's value and should be reported on a daily basis as part of the risk reporting process. For emerging economies with extreme volatility, the usage of stress-testing should be highly emphasized and full description of the process is included in any trading risk policy manual. Stress-testing usually takes the form of subjectively specifying scenarios of interest to assess changes in the value of the portfolio and it can involve examining the effect of past large market moves on today's portfolio. In this paper, risk management procedure is developed to assess potential exposure due to an event risk (severe crisis) that is associated with large movements of the GCC stock markets indices, under the assumption that certain GCC markets have typical 3%-12% leaps during periods of financial turmoil. The task here is to measure the potential trading risk exposure that is associated with a pre-defined leap and under the notion of several correlation factors and liquidation horizons.

4.2 Incorporating Liquidity Trading Risk Effects into L-VaR Modeling

Illiquid securities such as foreign exchange rates and equities are very common in emerging markets. Customarily these securities are traded infrequently (at very low volume). Their quoted prices should not be regarded as a representative of the traders' consensus vis-à-vis their real value but rather as the transaction price arrived at by two counterparties under special market conditions. This of course represents a real dilemma to anybody who seeks to measure the market risk of these securities with a methodology which is based on volatilities and correlation matrices. The main problem arises when the historical price series are not available for some securities, or even when they are available, they are not fully reliable due to the lack of liquidity.

Given that institutional investors usually have longer time horizons, the liquidity of their positions will be lower. The investment horizon of the investor as well as the liquidity characteristics of the mutual fund need to be integrated into the risk quantification process. For instance, portfolios with long investment horizons and/or low liquidity need specific risk measures in comparison to those that have shorter horizons and are very liquid. The choice of time horizon or number of days to liquidate (unwind) a position is a very important factor and has a strong impact on VaR numbers, and this time horizon depends upon the objectives of the portfolio, the liquidity of its positions and the expected holding period. Typically for a bank's trading portfolio invested in highly liquid currencies, a one-day horizon may be acceptable. For an investment manager with a monthly re-balancing and reporting focus, a 30–day period may be more appropriate. Ideally, the holding period should correspond to the longest period for orderly portfolio liquidation.

The simplest way to account for liquidity trading risk is to extend the holding period of illiquid positions to reflect a suitable liquidation period. An adjustment can be made by adding a multiplier to the VaR measure of each trading asset type, which at the end depends on the liquidity of each individual security. Nonetheless, the weakness of this method is that it allows for subjective estimation of the liquidation period. Furthermore, the typical assumption of a one-day horizon (or any inflexible time horizon) within VaR framework, neglects any calculation of trading risk related to liquidity effect (that is, when and whether a trading position can be sold out and at what price). A broad VaR model should incorporate a liquidity premium (or liquidity risk factor). This can be worked out by formulating a method by which one can unwind a position, not at some ad hoc rate, but at the rate when market conditions is optimal, so that one can effectively set a risk value for the liquidity effects. In general, this will raise significantly the VaR, or the amount of economic capital to support the trading position.

In fact, if returns are independent and they can have any elliptical multivariate distribution, then it is possible to convert the VaR horizon parameter from a daily to any *t*–day horizon. The variance of a *t*–day return should be *t* times the variance of a 1–day return or $\sigma^2 = f(t)$. Thus, in terms of standard deviation (or volatility), $\sigma = f(\sqrt{t})$ and the daily or overnight VaR number [*VaR (1-day)*] can be adjusted for any *t-day* horizon as:

$$VaR\ (t - day) = VaR\ (1 - day) * \sqrt{t} \tag{3}$$

The above formula was proposed and used by *J.P. Morgan* in their earlier *RiskMetrics™* method (1994). This methodology implicitly assumes that liquidation occurs in one block sale at the end of the holding period and that there is one holding period for all assets, regardless of their inherent trading liquidity structure. Unfortunately, the latter approach does not consider real-life-trading situations, where traders can liquidate (or re-balance) small portions of their trading portfolios on a daily basis. The assumption of a given holding period for orderly liquidation inevitably implies that assets' liquidation occurs during the holding period. Accordingly, scaling the holding period to account for orderly liquidation can be justified if one allows the assets to be liquidated throughout the holding period.

In this work we present a simple reengineered approach for calculating a closed-form parametric VaR with explicit treatment of liquidity trading risk. The proposed model and liquidity scaling factor is more realistic and less conservative than the conventional root-*t* multiplier. In essence the suggested multiplier is a function of a predetermined liquidity threshold defined as the maximum position which can be unwound without disturbing market prices during one trading day. The essence of the model relies on the assumption of a stochastic stationary process and some rules of thumb, which can be of crucial value for more accurate overall trading risk assessment during market stress periods when liquidity dries up. To this end, a practical framework of a methodology (within a simplified mathematical approach) is proposed below with the purpose of incorporating and calculating illiquid assets' daily VaR, detailed as follows:

The market risk of an illiquid trading position is larger than the risk of an otherwise identical liquid position. This is because unwinding the illiquid position takes longer than unwinding the liquid position, and as a result, the illiquid position is more exposed to the volatility of the market for a longer period of time. In this approach, a trading position will be thought of as illiquid if its size surpasses a certain liquidity threshold. The threshold (which is determined by each trader) is defined as the maximum position which can be unwound, without disrupting market prices, in normal market conditions and during one trading day. Consequently, the size of the trading position relative to the threshold plays an important role in determining the number of days that are required to close the entire position. This effect can be translated into a liquidity increment (or an additional liquidity risk factor) that can be incorporated into VaR analysis. If for instance, the par value of a position is \$10,000 and the liquidity threshold is \$5,000, then it will take two days to sell out the entire trading position. Therefore, the initial position will be exposed to market variation for one day, and the rest of the position (that is \$5,000) is subject to market variation for an additional day. If it assumed that daily changes of market values follow a stationary stochastic process, the risk exposure due to illiquidity effects is given by the following illustration, detailed along these lines:

In order to take into account the full illiquidity of assets (that is, the required unwinding period to liquidate an asset) we define the following:

t = number of liquidation days (t–days to liquidate the entire equity asset fully)

$\sigma_{adj}^{\ 2}$ = overnight (daily) variance of the illiquid equity trading position; and

σ_{adj} = liquidity risk factor or overnight (daily) standard deviation of the illiquid equity trading position.

The proposed approach assumes that the trading position is closed out linearly over t-days and hence it uses the logical assumption that the losses due to illiquid trading positions over t-days are the sum of losses over the individual trading days. Moreover, we can assume with reasonable accuracy that asset returns and losses due to illiquid trading positions are independent and identically distributed (*iid*) and serially uncorrelated day-to-day along the liquidation horizon and that the variance of losses due to liquidity risk over t-days is the sum of the variance (σ_i^2, for all $i = 1,2 \ldots, t$) of losses on the individual days, thus:

$$\sigma_{adj}{}^{2} = \left(\sigma_1{}^{2} + \sigma_2{}^{2} + \sigma_3{}^{2} + \cdots + \sigma_{t-2}{}^{2} + \sigma_{t-1}{}^{2} + \sigma_t{}^{2}\right) \quad (4)$$

In fact, the square root-*t* approach (equation 3) is a simplified special case of equation (4) under the assumption that the daily variances of losses throughout the holding period are all the same as first day variance, thus $\sigma_{adj}{}^{2} = \left(\sigma_1{}^{2} + \sigma_1{}^{2} + \sigma_1{}^{2} + \cdots + \sigma_1{}^{2}\right) = t\,\sigma_1{}^{2}$. As discussed above the square root-*t* equation overestimates asset liquidity risk since it does not consider that traders can liquidate small portions of their trading portfolios on a daily basis and then the whole trading position can be sold completely on the last trading day. Indeed, in real financial markets operations, liquidation occurs during the holding period and thus scaling the holding period to account for orderly liquidation can be justified if one allows the assets to be liquidated throughout the holding period. As such, for this special linear liquidation case and under the assumption that the variance of losses of the first trading day decreases linearly each day (as a function of *t*) we can derive from equation (4) the following:

$$\sigma_{adj}{}^{2} = \left((\frac{t}{t})^2 \sigma_1{}^{2} + (\frac{t-1}{t})^2 \sigma_1{}^{2} + (\frac{t-2}{t})^2 \sigma_1{}^{2} + \cdots + (\frac{3}{t})^2 \sigma_1{}^{2} + (\frac{2}{t})^2 \sigma_1{}^{2} + (\frac{1}{t})^2 \sigma_1{}^{2} \right) \quad (5)$$

Evidently, the additional liquidity risk factor depends only on the number of days needed to sell an illiquid equity position linearly. In the general case of *t*-days, the variance of the liquidity risk factor is given by the following mathematical functional expression of *t*:

$$\sigma_{adj}{}^{2} = f\left((\frac{t}{t})^2 + (\frac{t-1}{t})^2 + (\frac{t-2}{t})^2 + \cdots + (\frac{3}{t})^2 + (\frac{2}{t})^2 + (\frac{1}{t})^2 \right) \quad (6)$$

To calculate the sum of the squares, it is convenient to use a short-cut approach. From mathematical series the following relationship can be obtained:

$$(t)^2 + (t-1)^2 + (t-2)^2 + \cdots + (3)^2 + (2)^2 + (1)^2 = \frac{t(t+1)\,(2t+1)}{6} \quad (7)$$

Accordingly, after substituting equation 7 into equation 6 the liquidity risk factor can be expressed in terms of volatility (or standard deviation) as:

$$\sigma_{adj} = f\left\{ \sqrt{\frac{1}{t^2}[(t)^2 + (t-1)^2 + (t-2)^2 + \cdots + (3)^2 + (2)^2 + (1)^2]} \right\} \text{ or } \sigma_{adj} = f\left\{ \sqrt{\frac{(2t+1)\,(t+1)}{6t}} \right\} \quad (8)$$

The final result is of course a function of time and not the square root of time as employed by some financial market's participants based on the *RiskMetrics*^{TM} methodologies. The above approach can also be used to calculate the VaR for any time horizon. In order to perform the calculation of VaR under illiquid market conditions, the liquidity risk factor of equation 8 can be implemented in VaR calculation, hence, one can define the following:

$$L-VaR_{adj} = VaR\sqrt{\frac{(2t+1)(t+1)}{6t}} \tag{9}$$

VaR = Value-at-Risk under liquid market conditions.

$L\text{-}VaR_{adj}$ = Value-at-Risk under illiquid market conditions.

The latter equation indicates that $L\text{-}VaR_{adj} > VaR$, and for the special case when the number of days to liquidate the entire assets is one trading day, then $L\text{-}VaR_{adj} = VaR$. Consequently, the difference between $L\text{-}VaR_{adj} - VaR$ should be equal to the residual market risk due to the illiquidity of any asset under illiquid markets conditions. As a matter of fact, the number of liquidation days (t) necessary to liquidate the entire assets fully is related to the choice of the liquidity threshold; however the size of this threshold is likely to change under severe markets conditions. Indeed, the choice of the liquidation horizon can be estimated from the total trading position size and the daily trading volume that can be unwound into the market without significantly disrupting market prices; and in actual practices it is generally estimated as:

$$t = \textit{Total Trading Position Size of Asset}_i \,/\, \textit{Daily Trading Volume of Asset}_i \tag{10}$$

In real practice the daily trading volume of any trading asset is estimated as the average volume over some period of time, generally a month of trading activities. In effect, the daily trading volume of assets can be regarded as the average daily volume or the volume that can be unwound under a severe crisis period. The trading volume in a crisis period can be roughly approximated as the average daily trading volume less a number of standard deviations. Although this alternative approach is quite simple, it is still relatively objective. Moreover, it is reasonably easy to gather the required data to perform the necessary liquidation scenarios.

In essence, the above liquidity scaling factor (or multiplier) is more realistic and less conservative than the conventional root-t multiplier and can aid financial entities in allocating reasonable and liquidity market-driven regulatory and economic capital requirements. Furthermore, the above mathematical formulas can be applied for the calculation of VaR for each trading position and for the entire portfolio. In order to calculate the VaR for the full trading portfolio under illiquid market conditions ($L-VaR_{P_{adj}}$), the above mathematical formulation can be extended, with the aid of equation (2), into a matrix-algebra form to yield the following:

$$L-VaR_{P_{adj}} = \sqrt{\left| L-VaR_{adj} \right|^{T} * \left| \rho \right| * \left| L-VaR_{adj} \right|} \tag{11}$$

The above mathematical structure (in the form of three matrices, $\left| L - VaR_{adj} \right|$, $\left| L - VaR_{adj} \right|^{T}$ and $\left| \rho \right|$) can facilitate the financial-mathematical modeling and programming process so that the trading risk manager can specify different liquidation days for the whole portfolio and/or for each individual trading security according to the necessary number of days to liquidate the entire asset fully. The latter can be achieved by specifying an overall benchmark liquidation to liquidate the entire constituents of the portfolio fully. The number of days required to liquidate a position (of course, depending on the type of equity asset) can be obtained from the various publications of capital markets and can be compared with the assessments' of individual traders of each trading unit. As a result, it is possible to create simple statistics of the equity trading volume that can be liquidated and the necessary time horizon to unwind the whole volume. As a matter of fact, our modified liquidity risk factor approach, once compared with previously established liquidity risk models, could even lead to further reduction in the overall risk of the equity trading portfolio, and hence in the amount of regulatory capital and/or economic capital, as specified by Basel II requirements.

5. Measuring, Managing and Controlling of Market and Liquidity Risk Exposures — Empirical Relevance to Emerging GCC Financial Markets

In this study, the database of daily price returns of the six GCC stock markets' main indicators (indices) are gathered, filtered and adequately adapted for the creation of relevant inputs for the calculation of all risk factors. The historical database of daily indices levels is drawn from Reuters 3000 Xtra Hosted Terminal Platform. The total numbers of indices that are considered in this work are nine indices; seven local indices for the six GCC stock markets (including two indices for the UAE markets) and two benchmark indices, detailed as follows:

DFM General Index (United Arab Emirates, Dubai Financial Market General Index).

ADSM Index (United Arab Emirates, Abu Dhabi Stock Market Index).

BA All Share Index (Bahrain, All Share Stock Market Index).

KSE General Index (Kuwait, Stock Exchange Gener.al Index).

MSM30 Index (Oman, Muscat Stock Market Index).

DSM20 Index (Qatar, Doha Stock Market General Index).

SE All Share Index (Saudi Arabia, All Share Stock Market Index).

Shuaa GCC Index (Shuaa Capital, GCC Stock Markets Benchmark Index).

Shuaa Arab Index (Shuaa Capital, Arab Stock Markets Benchmark Index).

Moreover, in this work index returns are defined as $R_{i,t} = ln(P_{i,t}) - ln(P_{i,t-1})$ where $R_{i,t}$ is the daily return of index *i*, *ln* is the natural logarithm; $P_{i,t}$ is the current level of index *i*, and $P_{i,t-1}$ is the previous day index level. Furthermore, for this particular study we have chosen a confidence interval of 95% (or 97.5% with "one-tailed" loss side) and several liquidation time horizons to compute L-VaR. Historical database (of almost five years) of daily closing index levels, for the period 2004-2008, are assembled for the purpose of carrying out this research and further for the construction of market and liquidity risk management parameters and L-VaR risk limits.

In the process of analyzing the data as well as the empirical testing, firstly, the daily log returns of the nine indices are calculated. These daily returns are in fact essential ingredients for the calculation of standard deviations, correlation matrices, sensitivity factors (or beta coefficients), skewness, kurtosis and to perform the Jarque-Bera (JB) test of all the sample indices and their relationship vis-à-vis the Shuaa Arab Index. A software package (with an iterative optimization technique) is contrived for the purpose of creating realistic equity trading portfolios and consequently for carrying out the L-VaR's optimization of maximum trading limits under the notion of normal and extreme illiquid market conditions and by applying different correlation factors. The analysis of data and discussions of relevant findings and results of this research are organized and explained as follows:

5.1. Testing for Asymmetric Distributions of Assets Returns and Statistical Analysis of Volatility

In this section, analysis of the particular risk of each index (daily and annual volatility), the indices relationships with respect to the benchmark index (the Shuaa Arab Index) and finally a test of normality (symmetry) are performed on the sample equity indices. To investigate the statistical properties of the data, we have computed the log returns of each series. Table 1 illustrates the daily volatility of each of the sample indices under normal market and severe (crisis) market conditions. Crisis market volatilities are calculated by implementing an empirical distribution of past returns for all stock market indices' time series and hence, the maximum negative returns (losses), which are witnessed in the historical time series, are selected for this purpose. This approach can aid in overcoming some of the limitations of normality assumption and can provide a better analysis of VaR and especially under severe and illiquid market settings.

From Table 1 we can observe that the index with the highest volatility under normal market condition is the SE All Share Index whereas the DFM General Index demonstrates the highest volatility under severe market conditions. Annualized volatilities are depicted in Table 1, and this is performed by adjusting (multiplication) the daily volatilities with the square root of 260—assuming there are 260 trading days in the calendar year. An interesting outcome of the study of sensitivity factors (beta factors for systematic risk) is the manner in which the results are varied across the sample indices as indicated in Table 1. SE All Share Index appears to have the highest sensitivity factor (0.98) vis-à-vis the Shuaa Arab Index (which is the highest systematic risk) while the BA All Share Index seems to have the lowest beta factor (0.06). Moreover, and in accordance with general beliefs, Shuaa GCC Index (with a sensitivity factor of 1.05) is the best candidate of the entire sample indices that appears to move very closely with respect to the benchmark Shuaa Arab Index (with a beta factor of 1.0).

To take into account the distributional anomalies of asset returns, tests of normality (symmetry) are performed on the sample equity indices. In the first study, the measurements of skewness and kurtosis are achieved on the sample equity indices. The results are depicted in Table 2. It is seen, in general, that all indices have shown asymmetric behavior (both positive and negative values). Moreover, kurtosis studies have shown similar patterns of abnormality (i.e. peaked/flat distributions). At the upper extreme, MSM30 Index has shown a big negative skewness (-0.57) which is combined with a very high Kurtosis — a peak of (15.49). Some indices, such as in the case of DSM20 Index, has shown a close relationship to normality (Skewness of -0.11 and kurtosis of 2.61). As evidenced in Table 2, the above results of general departure from normality are also confirmed with the Jarque-Bera (JB) test. The JB statistics is calculated in this manner:

$$JB = n/6\left[S^2 + (K-3)^2/4\right] \approx \chi^2(2) \tag{12}$$

where S is the skewness, K is the kurtosis, and n is the number of observations. The JB statistics reassembles approximately a Chi-squared distribution [$\chi^2(2)$] with 2 degrees of freedom. The 95% and 99% percentage points of the Chi-squared distribution with 2 degrees of freedom are 5.99 and 9.21 respectively, thus, the lower the JB statistics, the more likely a distribution is normal. Nonetheless, the JB test shows an obvious general deviation from normality and, thus, rejects the hypothesis that GCC stock markets' time series returns are normally distributed. The interesting outcome of this study suggests the necessity of combining VaR calculations — which assumes normal distributions of returns — with other methods such as stress-testing and scenario analysis to get a detailed picture of other remaining risks (fat-tails in the probability distribution) that cannot be captured with the simple assumption of normality.

5.2. Matrices of Correlations and Analysis of Correlation Factors

Three matrices of correlations are created in this study, namely correlation = 1, 0, and empirical correlations. The objectives here are to establish the necessary quantitative infrastructures for advanced risk management analysis that will follow shortly. The assembled correlation matrix is depicted in Table 3, for the empirical correlation case of all nine indices. The latter correlation matrix is an essential element along with volatilities matrices for the creation of VaR and stress-testing calculations for equity market risk management processes and procedures. Contrary to general beliefs, our analysis indicates that there is a very small correlation (relationship) between the GCC stock markets in the long-run period. Nonetheless, in the short-run period (or on a daily crisis basis), however, we found that correlations tend to increase in value (although not on a large scale) and it could even switch signs under certain circumstances.

These long-run low correlation relationships are advantageous information for investors who would like to hold a diversified equity portfolio in GCC countries and particularly for medium/long investment horizon. In general, it seems that the Saudi market, with correlation factors of 62% and 60% respectively, dominates the panorama of actions and therefore has the biggest effect (and correlation relationship) on the Shuaa GCC and Shuaa Arab indices. The Dubai and Abu Dhabi financial markets have indicated a relatively moderate relationship of 56%. Likewise, and in accordance to general expectation, the Shuaa GCC and Shuaa Arab indices have shown a strong relationship of 93%.

5.3. Equity Trading Risk Management and Analysis with L-VaR Modeling

In order to illustrate the linkage between the theoretical constructs of L-VaR and its practical application and value as a tool for equity trading risk management, the following hypothetical trading portfolios with full case studies are presented. These case studies also help in understanding the methods used in determining the performance of alternative L-VaR estimation procedures in the context of equity trading risk management.

Using the definition of L-VaR in section 4 and under the assumption that a given equity portfolio has both long and short-selling trading positions, Tables 4 and 5 illustrate practical risk reports for the coverage of equity trading risk management activities of a hypothetical equity portfolio consisting of several indices of the GCC stock markets. Asset allocation and L-VaR analysis are performed under the assumption that local indices represent exact replicas of diversified portfolios of local stocks for each GCC stock market respectively. Furthermore, all risk analyses are performed at the one-tailed 97.5% level of confidence over different liquidation periods.

In the first full case-analysis study the total portfolio value is AED100 million (UAE Dirham) with different asset allocation percentage and one trading day liquidity horizon — that is, one day to unwind all equity trading positions. Furthermore, Table 4 illustrates the effects of stress testing (that is, L-VaR under severe market conditions) and different correlation factors on daily L-VaR calculations. The L-VaR-engine's report depicts also the overnight (daily) unconditional volatilities, which are calculated as the standard deviation of the percentage change in the index level (daily returns) of the nine indices, in addition to their respective sensitivity factors (or the beta factors) vis-à-vis the benchmark index. Crisis market daily volatilities (or downside-risk) are calculated and illustrated in the report. These daily severe downside-risk volatilities represent the maximum negative returns (losses), which are perceived in the historical time series, for all stock market indices. In essence, this approach can aid in overcoming some of the limitations of normality assumption and can provide a better analysis of (L-VaR) especially under severe and illiquid market settings. The effects of short-selling (albeit short-selling is currently not permitted in the GCC stock markets) are depicted in Table 4. One of the interesting results of this study is the way in which L-VaR numbers have decreased. This behavior might be explained by the way in which the overall portfolio is funded — in other words, long positions have been funded with short-selling of other stocks (or indices) and consequently have led to reduction in the overall risk exposure. In fact, one of the principal advantages of calculating L-VaR with matrix-algebra framework is the ability in which one can incorporate the effects of short-selling without tedious mathematical analysis.

The L-VaR modeling outputs are calculated under normal and severe market conditions by taking into account different correlation factors (empirical, zero and unity correlations between the various risk factors). Under correlation unity, one is assuming 100% positive relationships between all risk factors (risk positions) all the time, whereas for the zero-correlation case, there are no relationships between all positions at all times. The last correlation case takes into account the empirical correlation factors between all positions and is calculated via a variance/covariance matrix. Therefore, with 97.5% confidence, the actual equity trading portfolio should expect to realize no greater than AED 2,986,826 decrease in the value over a one-day time frame. In other words, the loss of AED 2,986,826 is one that an equity portfolio should realize only 2.5% of the time. If the actual loss exceeds the L-VaR estimate, then this would be considered a violation of the estimate. From a risk management perspective, the L-VaR estimate of AED 2,986,826 is a valuable piece of information. Since every equity trading business has different characteristics and tolerances toward risk, the trading risk manager must examine the L-VaR estimate relative to the overall position of the entire business. Simply put, can the firm tolerate or even survive such a rare event—a loss of AED 2,986,826 (or a 2.99% of total portfolio value)? This question is not only important to the equity trading unit, but also to financial institutions (or other funding units such as a treasury unit within the same hierarchy and organizational structure of the trading unit) who lend money to these trading units. The inability of a trading unit to absorb large losses may jeopardize their ability to make principal and interest payments. Therefore, various risk management strategies could be examined in the context of how they might affect the L-VaR estimate. Presumably, risk management strategies, such as the use of futures and options contracts in hedging possible fluctuation in equity prices, should reduce the L-VaR appraisal. In other words, those extreme losses in equity trading, that would normally occur only 2.5% of the time, should be smaller with the incorporation of some type of risk management strategy.

Furthermore, the analysis of L-VaR under illiquid market conditions is performed with three different correlation factors: empirical, zero and unity correlations respectively and for long and short trading positions. Indeed, it is essential to include different correlation factors in any L-VaR and stress-testing exercises. This is because existing trends in correlation factors may break down (or change signs) under adverse and severe market movements, caused by unforeseen financial or political crises. As expected, the case with correlation +1 provides the maximum VaR numbers (AED 4,176,532 and AED 25,089,744) as a result of the fact that under these circumstances total L-VaR of actual trading portfolio is the weighted average of individual L-VaRs of each equity trading position. Furthermore, the degree of risk-diversification (or in other words the effects of diversified L-VaR) of this hypothetical equity trading portfolio can also be deduced simply as the difference in the values of the two greatest L-VaRs — that is the L-VaR of correlation unity case versus the L-VaR of empirical correlation case (AED 1,189.706 or 39.83% for the normal market condition case). The overall sensitivity factor (beta factor) of this long/short equity portfolio is also indicated in this report as 0.745, or in other words, the total equity portfolio value, with actual asset allocation ratios, moves somehow in tandem with the benchmark index (Shuaa Arab Index). Moreover, Table 4 illustrates the individual risk factors for each equity index in terms of AED and under the notion of normal and severe market settings. These individual risk factors are in fact a reflection of a non-diversified L-VaR figures. Expected returns and risk-adjusted expected returns (under normal and severe market conditions) are also included in the L-VaR risk analysis report.

Finally, since the variations in L-VaR are mainly related to the ways in which the assets are allocated in addition to the liquidation horizon, it is instructive to examine the way in which L-VaR figures are influenced by changes in such parameters. All else equal, and under the assumption of normal and severe market conditions, Table 5 illustrates the non-linear alterations to L-VaR figures when the liquidation periods is increased in line and in accordance with the liquidity holding horizons as defined in Table 5 for all indices within the equity trading portfolio.

5.4. Optimization and Limit Setting for an Equity Trading Risk Management Unit Using L-VaR Modeling Technique

Optimization of maximum risk limits (or risk budgeting) are an important element for any equity trading risk management unit and it should be defined clearly and used wisely to ensure control on the trading/investment unit's exposure to risk. All limit-setting and control, monitoring and reporting should be performed by the risk management unit, independently from the front office's equity traders. In most professional trading and asset management units (such as commercial banks, foreign exchange dealers, commodity traders, institutional investors, etc.), VaR limit-setting is based on the concept of risk appetite. The risk appetite is defined as the maximum loss potential that management is willing to accept when an adverse move in equity prices occurs within a specified time horizon. In general, risk appetite will be dependent upon:

- The performance track-record in trading equities.
- The strategic importance of equity trading by the trading unit in question.
- The quality of the trading unit's infrastructure in handling the traded products.
- The overall exposure that the trading unit wants to have to proprietary trading and/or active investment risks in general and equity risks in particular.
- The volatilities of the equities (as determined by the risk manager) and the correlation factors between the different equity risk factors.
- Local and/or global regulatory constraints for the operations of equity securities.

How should we set risk limits to safeguard against maximum loss amounts? These are some of the central questions risk managers must envisage. In this paper a simplified, albeit practical approach is presented for the setting of optimized maximum L-VaR limits. To this end, an iterative optimization modeling technique with different L-VaR constraints has been examined in order to setup procedures for the establishment of maximum L-VaR limits. These maximum limits serve as a strict policy for handling situations in which the equity trading unit are above the authorized L-VaR limits. The L-VaR limits methodology and modeling procedure must be analyzed and approved by the board of directors of the equity trading entity. All trading/investment units need to have such limits of L-VaR as a guideline and also as a strict policy for their daily risk takings. Any excess of L-VaR beyond the ratified limits must be reported to top management by the risk management unit. Moreover, traders/asset managers need to give full and justified explanations of why their L-VaRs are beyond the approved limits.

Indeed, one of the basic problems of applied finance is the optimal selection of assets, with the aim of maximizing future returns and constraining risk by appropriate measure. To this end, Markowitz (1959) illustrated that, for given levels of risk, one can identify certain groups of equity securities that maximize expected returns. He considered these optimum portfolios as 'efficient' and referred to a continuum of such portfolios in dimensions of expected return and standard deviation as the efficient frontier. Accordingly, for asset allocation purposes, fund managers should choose portfolios located along the efficient frontier. Consequently, for more than four decades a wide body of knowledge has been accumulated about the performance, strengths, and weaknesses of this approach when applied to equity portfolios. However, much less is known about portfolio optimization techniques in emerging equity markets, particularly under illiquid and adverse market conditions.

In this research we look at the optimization problem from a different realistic operational angle. In view of that, the enigma is formulated by finding the portfolio that maximizes L-VaR, with expected return, trading volume and liquidation horizons that are constrained according to the requirements of the trading risk manager. As such, the focus in this work is on forecasting risk measures rather than anticipating expected returns for two reasons: first, several studies have analyzed the forecasts of expected returns in the context of mean-variance optimization (see for instance Best and Grauer, 1991). The common opinion is that expected returns are not easy to foresee, and that the optimization process is very sensitive to these variations. Second, there exists a general notion that L-VaR, in a wide sense, is simpler to assess than expected returns from historical data.

Essentially, our approach is a straightforward extension of the classic Markowitz mean-variance approach, where the original risk measure, variance, is replaced by L-VaR. The task is attained here by maximizing $L-VaR_{P_{adj}}$, while requiring a minimum expected return subject to several real-world operational constraints. For the purpose of this research and in order to ascertain coherent L-VaR trading limits the mathematical formulation for the optimization problem is formulated as follows

From equation 9 we can define liquidation horizon factor (LHF_i) for each trading asset as:

$$LHF_i = \sqrt{\frac{(2t_i+1)\,(t_i+1)}{6t_i}} \tag{13}$$

From equations 11 and 13, we can compute the maximum authorized L-VaR trading limits by solving for the following quadratic programming formulation:

$$Maximize \;\; L-VaR_{P_{adj}} = \sqrt{\left| \; L-VaR_{adj} \; \right| * \left| \; \rho \; \right| * \left| \; L-VaR_{adj} \; \right|^T} \tag{14}$$

Subject to the following budget constraints as specified by the risk manager:

$$\sum_{i=1}^{n} R_i x_i = R_P \;\; ; \; l_i \leq x_i \leq u_i \quad i = 1,2,....,n \tag{15}$$

$$\sum_{i=1}^{n} x_i = 1.0 \; ; \; l_i \leq x_i \leq u_i \quad i = 1,2,....,n \tag{16}$$

$$\left| \, LHF \, \right| \geq 1.0 \, ; \forall_i \;\; i = 1,2,.....,n \tag{17}$$

$$\sum_{i=1}^{n} V_i = V_P \qquad i = 1,2,....,n \tag{18}$$

Here R_P and V_P denote the target portfolio mean return and total portfolio volume respectively, and x_i the weight or percentage asset allocation for each asset. The values l_i and u_i, $i = 1, 2,\ldots, n$ denote the lower and upper constraints for the portfolio weights x_i. If we choose $l_i = 0$, $i = 1, 2,\ldots, n$, then we have the situation where no short selling is allowed. Moreover, $\left| LHF \right|$ indicates an $(nx1)$ matrix for all $i = 1, 2,\ldots, n$. The rationality of imposing the above constraints is to comply with current regulations which enforce regulatory capital requirements on investment companies, proportional to VaR and/or L-VaR of a trading portfolio besides other operational limits (for instance, volume trading limits).

In this study, the optimization process is based on the definition of L-VaR as the maximum possible loss over a specified time horizon within a given confidence level. The optimization modeling technique solves the problem by finding the market positions that maximize the loss, subject to the fact that all constraints are satisfied within their boundary values. Further, in all cases the liquidation horizons as indicated in Table 6 are assumed constant throughout the optimization process. For the sake of simplifying the optimization routine and thereafter its analysis, a volume trading position limit of 100 million AED is assumed as a constraint — that is the equity trading entity must keep a maximum overall market value of different trading positions of no more than 100 million AED (between long and short-selling positions). Furthermore, for optimization purposes, and in order to set up a more realistic risk management case, other constraints are imposed, detailed as follows:

- Total trading volume (of long and short trading positions) in all GCC stock markets is 100 million AED.
- Equity asset allocation is divided into 50% investment in Dubai, Abu Dhabi and Saudi stock markets and 50% in all other GCC financial markets.
- Total trading volume (of long and short trading positions) in Dubai, Abu Dhabi and Saudi stock markets is 50 million AED.
- Trading volume in any GCC stock market should be between [-40 and +40] million AED.
- All liquidity horizons for all equities are kept constant according to the specified values as indicated in Table 6.
- The overall portfolio expected return on a daily basis is set at a margin of 0.13%.

Now the weights are allowed to take negative or positive values, however, since arbitrarily high or low percentages make no financial sense, we propose to introduce a lower and an upper bound for the weights that are in accordance with reasonable trading practices. Full results of the optimization modeling process are given in Table 6. Based on the above optimization budget constraints and the results of Table 6, senior management of the financial trading/investment entity can set maximum daily L-VaR limits for the equity trading portfolio, detailed as follows:

- Maximum approved L-VaR limit under normal market conditions and with the assumption of empirical correlations = 3,330,779 AED.
- Maximum approved L-VaR limit under normal market conditions and with the assumption of positive 100% correlations = 3,347,040 AED.
- Maximum approved L-VaR limit under normal market conditions and with the assumption of zero correlations = 2,959,100 AED.

- Maximum approved L-VaR limit under severe or crisis market conditions and with the assumption of empirical correlations = 20,903,484 AED.
- Maximum approved L-VaR limit under severe or crisis market conditions and with the assumption of positive 100% correlations = 22,850,538 AED.
- Maximum approved L-VaR limit under severe or crisis market conditions and with the assumption of zero correlations = 18,603,366 AED.
- Maximum approved trading volume limit for all equities in the six GCC stock markets between long and short-selling positions = 100,000,000 AED.
- Maximum approved total asset allocation limit for Dubai, Abu Dhabi and Saudi stock markets between long and short-selling positions = 50,000,000 AED.
- Trading volume in any GCC stock market should be between [-40,000,000 and +40,000,000] AED and with the possibility of short-selling.
- The overall average portfolio expected return on a daily basis is set at a margin of 0.13%.
- The overall average portfolio risk-adjusted expected return on a daily basis and under the notion of normal market conditions is set at a margin of 3.9%.
- The overall average portfolio risk-adjusted expected return on a daily basis and under the notion of severe market conditions is set at a margin of 0.62%.

It should be noted here that the above optimum approved L-VaR trading limits are in their converted (or equivalent) UAE dirham (AED) values at the current or prevailing foreign exchange rates of other GCC countries versus the UAE dirham.

6. Summary and Concluding Remarks

This paper has presented a framework for calculating Liquidity-Adjusted Value at Risk (L-VaR) incorporating the liquidity of trading assets. In this work we proposed an enhanced L-VaR model which, unlike the standard version (root-*t* multiplier) that assumes that all trading positions can be sold instantaneously with no frication at the end of the holding period, takes into account different liquidation horizons with which the securities of a given portfolio could be liquidated. The key methodological contribution is the proposal of different and less conservative liquidity scaling factor for including liquidation risk in standard L-VaR analysis. The proposed liquidity multiplier is a function of a predetermined liquidity threshold, defined as the maximum position which can be unwound without disturbing market prices during one trading day, and is quite straightforward to implement even by very large financial institutions and institutional portfolio managers. As such, our framework facilitates the relatively simple liquidity-adjusted VaR under certain assumptions and recognizes liquidity trading risk as a significant risk factor that should be integrated within the framework of L-VaR. Furthermore, the model is theoretically simple with moderate demands on additional computing power while capturing the essential aspects of liquidity risk.

Equity trading risk management models, which are adopted in this work, are applied to the six GCC stock markets. Thus, our analyses are carried out for main market indicators in the GCC stock markets, in addition to two benchmark indices. To this end, database of daily indices closing levels (for the period 2004-2008) are obtained, filtered and matched for consistency of trading dates. Several case studies are carried out with the objectives of calculating L-VaR numbers under various scenarios and market conditions. The different scenarios are performed with distinct asset allocation percentages in addition to analyzing the effects of illiquidity of trading assets (unwinding horizon period of assets) and possibilities of short-selling. All analyses are carried out under the assumption of normal and severe (crisis) market conditions and under the notion of different correlation factors.

To investigate the statistical properties of the data, we have computed the log returns of each of the six stock markets index series. For almost all the cases, the study of some preliminary statistics allows us to conclude that the considered time series are characterized by asymmetry and high leptokurtosis. Moreover, the normality hypothesis has been rejected for almost every time series through the Jarque-Bera test. As a result, the use of normal distribution, which is the case in a mean-variance approach, tends to give poor evidence of what is observed in our return time series. In fact, L-VaR calculated under normality assumption can underestimate the actual risk exposure since the tails of the empirical distribution are fatter than those implied by the normal one. In order to overcome this shortcoming, in this work we implement the empirical distribution of past returns for all equity indices' time series. This approach has aided in providing a better analysis of L-VaR and especially under severe and illiquid market settings.

Our empirical testing results suggest that in almost all tests there are clear asymmetric behaviors in the distribution of returns of the sample equities. The appealing outcome of this study suggests the inevitability of combining L-VaR calculations with other methods such as stress-testing and scenario analysis to grasp a thorough picture of other remaining risks (such as, fat-tails in the probability distribution) that cannot be revealed with the plain assumption of normality. In conclusion, the implications of the findings of this study on the GCC stock markets suggest that although there is a clear departure from normality in the distribution of price returns, this issue can be tackled without the need for complex mathematical and analytical procedures. In fact, it is possible to handle these issues for cash equities with the simple use of variance/covariance method (in its matrix-algebra form) along with the incorporation of a credible stress-testing approach (under adverse market conditions) as well as by supplementing the risk analysis with a realistic liquidity risk factor that takes into account real-world trading circumstances. In this research, a reasonable model for the measurement of illiquidity of both short-selling and long trading positions is incorporated. In contrast to other commonly used liquidity models, the liquidity approach that is applied in this work is more appropriate for real-world trading practices since it considers selling small fractions of the long/short trading equity asset on a specific liquidation horizon. This liquidity factor can be implemented for the entire portfolio or for each equity asset in the trading portfolio.

Finally, L-VaR limits' setting is an important concern as part of the daily trading risk management process, and optimum risk limit structure should be brought into existence in any contemporary risk management procedure. To this end, an optimization modeling technique is developed to illustrate a practical approach for the setting of L-VaR limits for an equity-trading-unit. In all case studies, the volume limit in UAE dirham (AED100,000,000) is assumed constant and is used as a constraint (on the matrix-algebra's complex mathematical function) for the establishment of adequate and practical L-VaR limits. Furthermore, for optimization purposes, and in order to set up a more realistic risk management case, other asset allocation and portfolio expected return constraints are imposed. For this particular study, L-VaR limits are established for normal and severe market conditions and under the notion of different correlation factors. To this end, an iterative optimization and simulation technique is performed with different long/short asset allocation ratios and with the objectives of setting an optimum L-VaR limits structure for an equity trading risk management unit.

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Appendix: Tables of Relevant Statistical Analyses and Equity Risk Management Reports

Table (1) Risk Analysis Data: Daily and Annual Volatility and Sensitivity Factor

Stock Market Indices	Daily Volatility (Normal Market)	Daily Volatility (Crisis Market)	Annual Volatility (Normal Market)	Annual Volatility (Crisis Market)	Sensitivity Factor
DFM General Index	1.9%	12.2%	31.0%	196.0%	0.58
ADSM Index	1.4%	7.1%	22.8%	114.1%	0.40
BA All Share Index	0.6%	3.8%	9.5%	60.8%	0.06
KSE General Index	0.8%	3.7%	12.3%	60.2%	0.14
MSM30 Index	0.8%	8.7%	13.6%	140.3%	0.10
DSM20 Index	1.5%	8.1%	24.6%	130.2%	0.31
SE All Share Index	2.1%	11.0%	33.5%	177.9%	0.98
Shuaa GCC Index	1.4%	8.1%	23.4%	130.6%	1.05
Shuaa Arab Index	1.3%	7.6%	20.6%	122.1%	1.00

Table (2) Risk Analysis Data: Descriptive Statistics of Daily Returns, Skewness, Kurtosis and Jarque-Bera Test of Normality

Stock Market Indices	Maximum	Minimum	Median	Arithmetic Mean	Skewness	Kurtosis	Jarque-Bera (JB) Test
DFM General Index	9.9%	-12.2%	0.01%	0.12%	0.01	4.90	145**
ADSM Index	6.6%	-7.1%	0.00%	0.07%	0.12	4.29	69**
BA All Share Index	3.6%	-3.8%	0.00%	0.05%	0.43	7.28	769**
KSE General Index	5.0%	-3.7%	0.00%	0.09%	-0.18	5.42	241**
MSM30 Index	5.2%	-8.7%	0.00%	0.12%	-0.57	15.49	6340**
DSM20 Index	6.2%	-8.1%	0.00%	0.06%	-0.11	2.61	8*
SE All Share Index	9.4%	-11.0%	0.07%	0.03%	-0.97	5.51	407**
Shuaa GCC Index	11.1%	-8.1%	0.00%	0.06%	-0.66	11.06	2691**
Shuaa Arab Index	9.4%	-7.6%	0.00%	0.07%	-0.61	10.85	2549**

Note: Asterisks, * and **, denote statistical significance at the 0.05 and 0.01 levels respectively

Table (3) Risk Analysis Data: Correlation Matrix of Stock Market Indices

	DFM General Index	ADSM Index	BA All Share Index	KSE General Index	MSM30 Index	DSM20 Index	SE All Share Index	Shuaa GCC Index	Shuua Arab Index
DFM General Index	100%								
ADSM Index	56%	100%							
BA All Share Index	12%	8%	100%						
KSE General Index	17%	16%	12%	100%					
MSM30 Index	12%	17%	11%	11%	100%				
DSM20 Index	18%	23%	12%	12%	20%	100%			
SE All Share Index	20%	20%	7%	16%	11%	10%	100%		
Shuaa GCC Index	37%	35%	13%	19%	13%	26%	62%	100%	
Shuua Arab Index	39%	36%	12%	24%	15%	26%	60%	93%	100%

Table (4) Equity Trading Risk Management and Control Report (L-VaR Analysis, Full Case Study)

Asset Allocation and Liquidity-Adjusted Value at Risk (L-VaR) Report

Stock Market Indices	Market Value in AED	Asset Allocation	Liquidity Holding Horizon	Daily Volatility (Normal)	Daily Volatility (severe)	Sensitivity Factor	Expected Return	Individual Risk in AED (Normal)	Individual Risk in AED (Severe)
DFM General Index	$ 40,000,000	40.0%	1	1.93%	12.16%	0.58	0.12%	770,159	4,862,906
ADSM Index	$ 20,000,000	20.0%	1	1.42%	7.08%	0.40	0.07%	283,118	1,415,107
BA All Share Index	$ (20,000,000)	-20.0%	1	0.59%	3.77%	0.06	0.05%	117,822	753,631
KSE General Index	$ (20,000,000)	-20.0%	1	0.76%	3.74%	0.14	0.09%	152,255	747,293
MSM30 Index	$ 20,000,000	20.0%	1	0.84%	8.70%	0.10	0.12%	168,398	1,739,797
DSM20 Index	$ 20,000,000	20.0%	1	1.53%	8.07%	0.31	0.06%	305,688	1,614,827
SE All Share Index	$ 40,000,000	40.0%	1	2.08%	11.03%	0.98	0.03%	830,979	4,413,158
Shuaa GCC Index	$ -	0.0%	1	1.45%	8.10%	1.05	0.06%	-	-
Shuaa Arab Index	$ -	0.0%	1	1.28%	7.57%	1.00	0.07%	-	-
Total Portfolio Value in AED	$ 100,000,000	100%					0.08%		

Expected Return and Risk-Adjusted Return

Trading Portfolio Expected Return	0.08%
Risk-Adjusted Expected Return (Normal)	2.78%
Risk-Adjusted Expected Return (Severe)	0.47%

Daily Liquidity-Adjusted Value at Risk (L-VaR) in AED [Normal Market Conditions]

Correlation = Empirical	Correlation = 1	Correlation = 0
2,986,826	4,176,532	2,467,949
2.99%	4.18%	2.47%

Diversification Benefits	
$ 1,189,706	39.83%

Daily Liquidity-Adjusted Value at Risk (L-VaR) in AED [Severe (Crisis) Market Conditions]

Correlation = Empirical	Correlation = 1	Correlation = 0
17,496,243	25,089,744	14,406,571
17.50%	25.09%	14.41%

Diversification Benefits	
$ 7,593,500	43.40%

Overall Sensitivity Factor: Portfolio of Stock Indices
0.745

Table (5) Equity Trading Risk Management and Control Report (L-VaR Analysis, Full Case Study)

Asset Allocation and Liquidity-Adjusted Value at Risk (L-VaR) Report

Stock Market Indices	Market Value in AED	Asset Allocation	Liquidity Holding Horizon	Daily Volatility (Normal)	Daily Volatility (severe)	Sensitivity Factor	Expected Return	Individual Risk in AED (Normal)	Individual Risk in AED (Severe)
DFM General Index	$ 40,000,000	40.0%	2	1.93%	12.16%	0.58	0.12%	861,064	5,436,895
ADSM Index	$ 20,000,000	20.0%	2	1.42%	7.08%	0.40	0.07%	316,536	1,582,138
BA All Share Index	$ (20,000,000)	-20.0%	3	0.59%	3.77%	0.06	0.05%	146,949	939,944
KSE General Index	$ (20,000,000)	-20.0%	3	0.76%	3.74%	0.14	0.09%	189,895	932,038
MSM30 Index	$ 20,000,000	20.0%	4	0.84%	8.70%	0.10	0.12%	230,588	2,382,316
DSM20 Index	$ 20,000,000	20.0%	4	1.53%	8.07%	0.31	0.06%	418,581	2,211,193
SE All Share Index	$ 40,000,000	40.0%	2	2.08%	11.03%	0.98	0.03%	929,063	4,934,061
Shuaa GCC Index	$ -	0.0%	1	1.45%	8.10%	1.05	0.06%	-	-
Shuaa Arab Index	$ -	0.0%	1	1.28%	7.57%	1.00	0.07%	-	-
Total Portfolio Value in AED	$ 100,000,000	100%					0.08%		

Expected Return and Risk-Adjusted Return

Trading Portfolio Expected Return	0.08%
Risk-Adjusted Expected Return (Normal)	2.43%
Risk-Adjusted Expected Return (Severe)	0.41%

Daily Liquidity-Adjusted Value at Risk (L-VaR) in AED [Normal Market Conditions]

Correlation = Empirical	Correlation = 1	Correlation = 0
3,421,759	4,837,975	2,821,927
3.42%	4.84%	2.82%

Diversification Benefits	
$ 1,416,215	41.39%

Daily Liquidity-Adjusted Value at Risk (L-VaR) in AED [Severe (Crisis) Market Conditions]

Correlation = Empirical	Correlation = 1	Correlation = 0
20,190,327	29,349,241	16,580,100
20.19%	29.35%	16.58%

Diversification Benefits	
$ 9,158,914	45.36%

Overall Sensitivity Factor: Portfolio of Stock Indices
0.745

Table (6) Equity Trading Risk Management and Control Report (L-VaR Limits Setting)

(Optimization Technique Outcomes)

Asset Allocation and Liquidity-Adjusted Value at Risk (L-VaR) Report

Stock Market Indices	Market Value in AED	Asset Allocation	Liquidity Holding Horizon	Daily Volatility (Normal)	Daily Volatility (severe)	Sensitivity Factor	Expected Return	Individual Risk in AED (Normal)	Individual Risk in AED (Severe)
DFM General Index	$ 40,000,000	40.0%	2	1.93%	12.16%	0.58	0.12%	861,064	5,436,895
ADSM Index	$ 40,000,000	40.0%	2	1.42%	7.08%	0.40	0.07%	633,072	3,164,276
BA All Share Index	$ (40,000,000)	-40.0%	3	0.59%	3.77%	0.06	0.05%	293,899	1,879,887
KSE General Index	$ 29,520,768	29.5%	3	0.76%	3.74%	0.14	0.09%	280,293	1,375,724
MSM30 Index	$ 40,000,000	40.0%	4	0.84%	8.70%	0.10	0.12%	461,176	4,764,632
DSM20 Index	$ 20,479,232	20.5%	4	1.53%	8.07%	0.31	0.06%	428,611	2,264,176
SE All Share Index	$ (30,000,000)	-30.0%	2	2.08%	11.03%	0.98	0.03%	696,797	3,700,546
Shuaa GCC Index	$ -	0.0%	1	1.45%	8.10%	1.05	0.06%	-	-
Shuaa Arab Index	$ -	0.0%	1	1.28%	7.57%	1.00	0.07%	-	-
Total Portfolio Value in AED	$ 100,000,000	100%					0.13%		

Expected Return and Risk-Adjusted Return

Trading Portfolio Expected Return	0.13%
Risk-Adjusted Expected Return (Normal)	3.90%
Risk-Adjusted Expected Return (Severe)	0.62%

Daily Liquidity-Adjusted Value at Risk (L-VaR) in AED [Normal Market Conditions]

Correlation = Empirical	Correlation = 1	Correlation = 0
3,330,779	3,347,040	2,959,100
3.33%	3.35%	2.96%

Diversification Benefits	
$ 16,261	0.49%

Daily Liquidity-Adjusted Value at Risk (L-VaR) in AED [Severe (Crisis) Market Conditions]

Correlation = Empirical	Correlation = 1	Correlation = 0
20,903,484	22,850,538	18,603,366
20.90%	22.85%	18.60%

Diversification Benefits	
$ 1,947,054	9.31%

Overall Sensitivity Factor: Portfolio of Stock Indices
0.222