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Abstract

This paper builds a theoretical model based on Allen and Gale (2000) to analyze how an unexpected shock affecting the banking assets in one region can generate bankruptcy in a second region. I also analyze the effect of the presence of an Islamic bank in a third region on the vulnerability of conventional banks to financial contagion. It is shown that the Islamic bank's assets diversification strategy across the regions reduces the likelihood of financial contagion among conventional banks.

JEL Classification: G01, G21

Keywords: Islamic Banking, Conventional Banking, Financial Crisis

ملخص

تبني هذه الورقة نموذج نظري يعتمد على ألين وغيل (2000) لتحليل كيف يمكن لصدمة غير متوقعة ان تؤثر على الأصول المصرفية في منطقة واحدة وكيف يمكن أن تولد الإفلاس في المنطقة الثانية. وأود أيضا تحليل أثر وجود بنك إسلامي في منطقة ثالثة على ضعف البنوك التقليدية والعدوى المالية. ويظهر أن استراتيجية التنويع في مختلف المناطق لأصول البنوك الإسلامية يقلل من احتمالات العدوى المالية بين البنوك التقليدية.

1. Introduction

Islamic banks (IBs) assets have increased globally to reach US$826 billion in 2010 and are projected to reach US$ 1,130 billion in 2012 (Ernst & Young 2011). This tendency is also perceivable at the country levels where the share of the Islamic banks' assets accounts for 14% in the Middle East and North Africa (MENA) region, for 26% in the Gulf Cooperation Council (GCC) and for 17.3% in Malaysia. Therefore, the banking system is becoming dual in many countries where there is a presence of both conventional and Islamic banks. This duality generated a new challenge for central banks in designing their regulatory and supervisory frameworks. It also stimulated studies trying to compare the behavior and performance of the two types of banks. The empirical studies revealed that the current practices of IBs deviate from their theoretical model as the majority of IBs mimic the conventional banks' (CBs) business model by creating assets through debt-like instruments with a predetermined fixed rate of return. On average almost 80% of the IB's total assets are of fixed income and short-term maturity. While, only 20% are long term and risk sharing investments.

El-Hawary el al. (2007) and Greuning and Iqbal (2008) claim that the dominance of less risky, low return assets (e.g. cost-plus sale (*Murabahah)* and leasing (*Ijarahs)*) deprive IBs of the benefits of portfolio diversification, as profit sharing and loss bearing (*Mudarabah)* and profit and loss sharing *Musharakah* contracts are more profitable. According to Siddiqi (2006) this behavior could be explained by the low moral hazard and adverse selection problems associated with sale-based transactions compared to the profit and loss sharing (PLS) investments. Another salient divergence with the Islamic banking theory is also revealed by the income distribution. In many cases, IBs distribute profits to the investment depositors even when they accrue loss (El-Hawary et al. 2007; Greuning and Iqbal 2008). This displaced commercial risk was confirmed by Zainol and Kassim (2010) and Cevik and Charap (2011) who found that the CBs' deposit rates Granger cause returns on PLS accounts in Malaysia and Turkey. A comparable result was established by Chong and Liu (2009) in the case of Malaysia, showing that the retail Islamic deposit rates mimic the behavior of conventional interest rates. Beck et al. (2010) carried an empirical investigation on a broad cross-country sample and reached the same conclusion since they identified few significant differences in business orientation, efficiency, asset quality between conventional and Islamic retail banks. The stability of IBs—relative to CBs—was analyzed by Cihak and Hesse (2010), which found that IBs are stronger only when they are small in size. They also found no positive impact of the IBs' presence on the financial strength of CBs.

The analysis of the stability of IBs relative to CBs becomes more relevant when period under analysis contains within it the recent global financial crisis. Indeed, the crisis induced a series of CB failures and constitutes a good test for IBs' stability. From a theoretical perspective, the PLS principle enables the IBs to maintain their net worth under difficult economic situations. Indeed, any shock that could generate losses on their asset side would be absorbed on the liability side. (Ahmed 2002; Cihak and Hesse 2010). However, a growing number of studies show that the recent crisis has impacted not only the CBs but also IBs. Hasan and Dridi (2010) had reached the same conclusion using a sample of 120 Islamic and conventional banks in 8 countries. Beck et al. (2010) showed that conventional banks operating in countries with a high market share of Islamic banks are more cost-effective but less stable. Besides, their results confirmed the relatively better performance of IBs during the recent crisis. According to Syed Ali (2007) *Ihlas Finance House*, the Turkish Islamic financial institution collapsed and shut down during the financial crisis of 2000-2001 due to liquidity problems and financial distress that originated from its strategic error to allow withdrawals from Investment Accounts. "On the contrary other SFHs (Islamic financial institutions) which survived the crisis did not en-cash the investment deposits and advised their clients to hold them to maturity."[1]

The objective of this paper is to shed light on another angle of the interaction between IBs and CBs. More precisely, I analyze the behavior of an IB when bankruptcy occurs in the conventional banking sector and investigate whether the presence of the IB reduces the likelihood of financial contagion within CBs[2].

One of the justifications of the existence of banks in the conventional "fractional reserve system" is their role as "pools of liquidity" providing depositors with insurance against idiosyncratic shocks that affect their consumption needs (Freixas and Rochet 2008). In this system, banks hold a fraction of their deposits as cash reserve and the rest of the deposits are used to finance profitable but illiquid investments. When offering demand deposit contracts, a bank becomes inherently vulnerable to bank run which takes place when all depositors panic and withdraw their deposits immediately irrespective of their consumption (liquidity) needs. This liquidity shock (excess of the immediate demand for liquidity which obliges the bank to liquidate its long assets since its short assets are insufficient) has been considered the triggering event in the banking crisis theory that focused on contagion among banks[3]. Allen and Gale (2000) show that an unforeseen liquidity shock could generate the bankruptcy of the entire banking system under different configurations of the interbank market structure. In Allen and Gale (2000), banks belonging to different regions are cross-holding deposits on the interbank market. The authors show that the completeness of the latter (each bank is connected to all banks) reduces the likelihood of transmission of a bankruptcy from one region to another. The opposite happens when the interbank market is incomplete (each bank is connected to a few number of banks). This is because in the first configuration each bank in the different untroubled regions liquidates a small amount of its long asset and responds to the liquidity needs of the troubled bank without suffering bankruptcy.

In Freixas et al. (2000) a liquidity shock that hits one of the major banks may prompt depositors to make a run on solvent banks if they fear that there is insufficient liquidity in the banking system. However, as stated by Adrian and Shin (2008) "the domino model (of contagion) paints a picture of passive financial institutions that stand by and do nothing as the sequence of defaults unfolds." In practice, financial distresses are more likely to be triggered by the impact of price changes on the banks' balance sheets. Mishkin (1998) affirms that the deterioration in bank balance sheets could result from excessive risk-taking due to inadequate bank regulation and supervision or because of negative shocks such as interest rate rises or stock market crashes, among other factors. This is confirmed by the 2007 subprime crisis which originated as a relatively small credit default event[4]. During this crisis the difficulty to access funds was coupled with the decline of the prices of the structured mortgage assets, thus weakening the balance sheets of many banks exposed to the U.S. asset-backed securities (Hesse et al. 2008). As a consequence, major investments banks like Lehman Brothers went bankrupt while others like Northern Rock and Bear Stearns were rescued.

This paper builds a theoretical model that analyzes the effect of the presence of an IB on the propagation of bankruptcy across two CBs. We consider that the CBs belong to two different

[1] Syed Ali (2007, 12).

[2] Financial contagion between banks occurs when the bankruptcy of one bank causes the bankruptcy of a second bank or groups of banks (Allen and Gale 2000).

[3] The term "liquidity" is linked here to the concept of "funding liquidity" which refers to the availability of funds and the ability of a solvent bank (a financial institution in general) to perform its intermediation function. The other concept is "market liquidity" which refers to the ease with which positions may be traded without significantly affecting their corresponding asset price (Crockett 2008; Hesse et al. 2008).

[4] "A one percent gain or loss in the US stock market is about the same order of magnitude of the likely subprime mortgage losses that will be gradually realized over the next few years." (Adrian and Shin 2010, 2)

regions and that the IB belongs to a third region. Each region is characterized by different investment opportunities available in different industries and populated by a continuum of consumers (depositors) of mass 1. The interconnectedness between the regions takes the form of direct investments (purchasing of equity shares) by banks of one region in the investment projects of the other region. When a bankruptcy occurs in one region, the other region is impacted due to the premature termination (liquidation) of the investment projects financed by the bankrupted bank. We consider that this bankruptcy could generate a negative externality in the form of reduced return of the remaining long investment projects. In practice, in case of a banking crisis, banks are likely to cut on their lending in order to shrink their asset base and restore their capital ratio. Since all businesses rely on finance to function, the economic activity will slow[5] and the economies of scale will most likely decline reducing the return of the investment activities. In our model we enable different levels of the negative externality affecting the return of the long investment projects. This externality affects the returns of the IB without causing its bankruptcy due to the amortizing effect of the PLS investment accounts. In addition, we show that the IB is incited to use its liquid funds to reduce the premature liquidation of the long investment projects by acquiring—at a discount—the shares of the investment projects initially owned by the bankrupt conventional bank. This in turn limits the negative effect on the bankruptcy externality on the second CB and reduces the likelihood of its financial contagion. To my knowledge, this is the first theoretical attempt that analyzes this type of interaction between IBs and CBs.

The rest of the paper is organized as follows. Section 2 develops a model of financial contagion within a banking system comprising only two conventional banks. Section 3 assesses the effect of the IB's presence on the propagation of bankruptcy among the CBs. Finally, section 4 summarizes the main results.

2. A Model of Simultaneous Bankruptcy of Two Conventional Banks

In this section, we develop a theoretical model based on Allen and Gale (2000) to analyze how unexpected shocks affecting the banking assets in one region can generate simultaneous bankruptcy of the CBs across two different regions. Many features distinguish our model from the above mentioned ones. For example we consider inter-banking linkages through the financing of common investment projects instead of cross-holding deposits through the interbank money market as is the case in Allen and Gale (2000). Besides, the source of the financial fragility is an unexpected productivity shock[6] affecting the investment project instead of the liquidity shock. Finally, we show that a decrease in the cost of premature termination of the investment project (liquidation cost) increases the vulnerability to simultaneous bankruptcy of the banks in the two regions. This result is opposite to that obtained by Allen and Gale (2000) in the context of a liquidity shock that propagates through the interbank deposit market.

2.1 The Economic environment

There are three dates $t = 0, 1, 2$. There is a single consumption good and the economy is divided in two regions labeled A and B which can be interpreted as geographical regions with particular specialized sectors within a country. Each region contains a competitive banking sector which could be represented by a single bank and a continuum [0,1] of identical consumers (depositors).

[5] The investment spending and economic activity may remain depressed for a long period in industrialized countries due to the "debt deflation" (Mishkin 1998; Bhattacharya et al. 1998). For example this was the case of Japan in the early 1990s and the U.S. in 1930-1933.

[6] This unforeseen exogenous productivity could be triggered by a political instability which worsens the macroeconomic environment.

2.1.1 Investment opportunities

In each region banks have two types of investment opportunities. First, there is a storage that yields a unit safe return. Indeed, one unit of the consumption good invested in this short investment at date *t* produces one unit of the consumption good at date *t+1*. Second, there is a long-term investment project that has a higher expected return but matures after two periods and yields the following stochastic gross return over the two periods with $R_H > 1 > R_L$ and $E[R] = R_a = (R_L + R_H)/2 > 1$ signifying that the long-term asset is more productive on average than the short-term asset. If R_H occurs, the economy is in the high state of the nature S_H otherwise it is in the low state S_L.

$$R = \begin{cases} R_H & \text{with probability } 1/2 \\ R_L & \text{with probability } 1/2 \end{cases} \tag{1}$$

Definition 1. The "liquidation cost" is the cost of premature termination of the investment project.

If the owner of the investment project is asked by his financiers to repay their capital while the production process has not been completed it is natural that this will generate additional costs for the firm (e.g. the project owner will be obliged to borrow or to sell part of the project's raw material/equipment) and generate lower cash flow. Another justification of the liquidation cost rests on the fact that in the presence of economies of scale (which is the case of many industries) the premature liquidation of a part of the investment project reduces its size and decreases its productivity. Therefore, the long-term asset is not completely illiquid and each unit of the long-term asset can be prematurely liquidated to produce rR units of the consumption good at *date 1 (*where *R* is given by 1*)* such that $r<1$ and $rR_a < rR_H < 1$.

2.1.2 Economic signals

All uncertainty is resolved at date 1 when banks and depositors observe a signal $S \in \{S_1, S_2\}$ that predicts with perfect accuracy the state of the nature that will take place in regions A and B at date 2. As in Allen and Gale (1998) this signal can be thought of as a leading economic indicator that predicts the value of the investment projects' cash-flows in each region. Although, there is an uncertainty at date *0* regarding the states of the nature that will occur at date 1, its distribution is known by all banks and depositors and given by Table 1.

Hence in region *i=A, B* there is a probability of ½ that the state of the nature S_H occurs and a probability of ½ that the state S_L occurs. It is clear that given this distribution of the states of nature, the average gross return of the investment opportunity across the economy (comprising the two regions) is certain and equals R_a in each state[7].

2.1.3 Banks' inter-linkages

In order to diversify their assets banks will invest in the short-term investment as well as in the two regions' long-term investment projects. Banks invest by acquiring equity shares issued by the projects' owners. This direct investment in the region was discussed in Allen and Gale (2000) when they argued that their model could be extended to the case of risky long-term asset[8]. This diversification reduces the risk of each bank's long-term investment

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[7] In each region there is an investment of 1 unit of the consumption good. Therefore the total investment across the economy (the two regions) is equal to 2 units. In case of S_1 or S_2 the total output across the two regions is certain and equal to $R_H + R_L$. The uncertainty (which is released at t = 1) concerns which region will contribute with R_H and which one will contribute with R_L. Therefore for two units invested at t = 0 the economy produces $R_H + R_L$ at date t = 2. It follows that the average return is $(R_H + R_L)/2 = R_a$.

[8] The authors argued that their results will remain the same if the financial interconnectedness between the regions takes the form of claims held by banks in one region on banks in another region. However, they

since it guarantees a certain average gross return of R_a. In the absence of this diversification the gross return is random and may be equal to R_H or R_L depending on the state of the nature that occurs in the bank's region.

In addition, it is possible that the liquidation of the long investment project by one bank may reduce the return of all investment projects even if they are continued till their maturity. In this case, the return of the long-term investment project becomes φR instead of R where $r < \varphi \leq 1$. If φ equals one there is no negative externality. In conclusion, the liquidated investment project generates no cash flows at date 2 and rR at date 1. Whereas, the remaining investment project of the region generates the following cash-flows at date 2

$$\varphi R = \begin{cases} \varphi R_H & \text{with probability } 1/2 \\ \varphi R_L & \text{with probability } 1/2 \end{cases}$$

2.1.4 Depositors (consumers)

Each region contains a continuum of mass 1 of ex-ante identical consumers (depositors). A consumer has an endowment equal to one unit of the consumption good at date 0 and nothing at dates 1 and 2. Consumers are initially uncertain about their time preferences. At date 1 each consumer knows whether he is an *early consumer* who only want to consume at date 1 or *late consumer* who only want to consume at date 2. In addition, this is private information for the consumer which is not observable by banks. Hence, late consumers can pretend to be early consumers and withdraw their deposits at date 1 if they will obtain a higher return than withdrawing at date 2. It is only in section 2.2 that we assume that a social planner can identify the type of each consumer. At date 0 each consumer has a probability γ to be an early consumer and a probability 1 - γ to be a late consumer. Therefore, the ex-ante preferences of a consumer could be represented by

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{with probability } \gamma \\ \delta u(c_2) & \text{with probability } 1\text{-}\gamma \end{cases}$$

Where c_t denotes consumption at date t =1, 2 and $\delta < 1$ is the discount factor. The utility function *u(.)* is assumed to be twice continuously differentiable, increasing, and strictly concave. In ex-ante terms the expected utility of a consumer is

$$EU = \gamma Eu(c_1) + (1-\gamma)\delta Eu(c_2) \tag{2}$$

2.2 Autarky

We start by the simplest case where each consumer chooses independently the quantity y that he invests in the short asset and $x = 1 - y$ that he invests in the long investment project. If he has to consume early then the quantity x should be liquidated at date 1. Therefore, we have

$$\begin{aligned} c_1 &= y + rRx \\ c_2 &= y + Rx \end{aligned} \tag{3}$$

Where R is the random cash flow of the investment project given by (1) which is revealed at date 1. Initially at date 0 the consumer chooses the allocation (x^a, y^a) so as to maximize its

mentioned that "*If, instead of holding claims on banks in other regions, banks were to invest directly in the long assets of that region, there would be a spillover effect, but it would be much weaker.*" (Allen and Gale 2000, 31).

expected utility EU under the constraints (3). At date 1 all the uncertainties are released and we have the following cases

	Early Consumer	**Late Consumer**
High return R_H	$c_1^a = y^a + rR_H x^a < 1$ $c_2^a = 0$	$c_1^a = 0$ $c_2^a = y^a + R_H x^a$ with $y < c_2^a < R_H$
Low return R_L	$c_1^a = y^a + rR_L x^a < 1$ $c_2^a = 0$	$c_1^a = 0$ $c_2^a = y^a + R_L x^a < 1$

Since $rR_L < rR_H < 1$ and $R_L < 1$ the allocation (x^a, y^a) will be ex post (at date 1) suboptimal in all the cases. Indeed, if the consumer is an early one the optimal decision is $(x, y) = (0,1)$ and the consumption is $c_1 = 1 > c_1^a$. While, if the consumer is of a late type and the return is high the optimal decision is $(x, y) = (1,0)$ and the consumption is $c_2 = R_H > c_2^a$. When the consumer is of a late type and the return is low the optimal decision is $(x, y) = (0,1)$ and the consumption is $c_2 = 1 > c_2^a$. This inefficiency can be mitigated by a social planner who maximizes the expected utility of the entire population of consumers overs the economy (two regions).

2.3 The Optimal (symmetric) Allocation

We now consider that there is a social planner that collects the two units of consumption good endowments of the consumers across the two regions. He invests $2y$ units in the short asset and x in the long investment project of region A and x in the long investment project of region B. Therefore, the total quantity of the consumption good available at date 1 is $2y$ whereas it is $R_H x + R_L x = 2R_a x$ at date 2. The social planner maximizes the following social expected utility $EU = 2\gamma Eu(c_1) + 2(1-\gamma)\delta Eu(c_2)$ which is the sum of the consumers expected utilities. The parameter γ (respectively 1 - γ) represents the probability for an individual to be an early (late) consumer. Since the total mass of consumers in the economy is equal to 2 and using the law of large numbers, the probability 2γ (respectively $2(1-\gamma)$) represents also the fraction of early (respectively late) consumers in the economy. Therefore, the social planner program is given by

$$\begin{cases} \underset{(x,y)}{Max}\ EU = 2\gamma Eu(c_1) + 2(1-\gamma)\delta Eu(c_2) & (4) \\ s.t. & \\ 2x + 2y = 2 & (5) \\ 2\gamma c_1 \le 2y & (6) \\ 2(1-\gamma)c_2 = 2R_a x + (2y - 2\gamma c_1) & (7) \end{cases}$$

Recalling equation (2) it is clear that maximizing the objective function (4) is equivalent to maximizing the expected utility of each individual consumer in the economy belonging to region A or B (which are ex-ante identical). Constraint (5) means that the value of the total investment equals to the available funds. Constraint (6) signifies that the consumption needs of the early consumers 2γ are covered by the investment $2y$ in the short asset. Constraint (7) signifies that the output $R_a x$ from the investment projects plus the residual quantity from the short investment after the payment of early consumers equals the consumption needs of the late consumers. It is simple to show[9] that the solution of the planning problem is characterized by the following conditions:

$$\begin{cases} u'(c_1^*) = \delta R_a u'(c_2^*) \\ c_1^* = y^* / \gamma \\ c_2^* = R_a x^* / (1-\gamma) \end{cases} \qquad \begin{matrix} (8) \\ (9) \end{matrix}$$

Since the utility function u is concave the late consumer obtains higher amount than the early consumer $c_2^* \geq c_1^*$ if and only if [10]

$$\delta R_a \geq 1 \qquad (10)$$

Under this condition a late consumer has no incentive to declare that he is an early one to obtain c_1 and stores it to consume at date 2. Therefore, even if the planner cannot observe the consumers' types each of the consumers will correctly reveal it. Thus, the above characterized optimal allocation can be achieved in this more general case.

Lemma 1.

The optimal allocation dominates the autarky allocation. (for the proof, see the appendix).

By pooling the deposits of a large number of consumers, the social planner can offer them insurance against their uncertain consumption needs. This is done by providing early consumers with some of the high yielding risky asset without exposing them to the volatility of the investment projects in their region.

2.4 Decentralization of the Optimal Allocation

In each region there is a continuum of banks constituting a competitive banking system. Banks are assumed to be identical and adopt the same behavior which simplifies the analysis by considering a representative bank for each region. Each consumer deposits his endowment of one unit of the consumption good in the representative bank of his region in exchange for a contract (c_1^*, c_2^*) (which is the solution of the planning problem) allowing him to withdraw either c_1^* units of consumption at date 1 or c_2^* units consumption at date 2. In the rest of the paper we denote (c_1^*, c_2^*) by (c_1, c_2). Since the deposit contract is not contingent on the state of nature that will occur in regions A and B the question that arises is how banks of the two regions can perform the role played by the social planner in the previous section. This is done through investing in the long-term investment projects in regions A and B[11]. Indeed, the bank belonging to region $i=A,B$ invests its one unit of deposits in a portfolio (x^i, y^i, z^i) where x^i and

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[9] Constraints (6) and (7) hold with equality since it is not optimal to invest in the short asset above the consumption needs at date 1, i.e. $y = \gamma c_1$. This is because it is possible to invest in the investment projects across the economy and obtain a higher certain gross return Ra>1. Hence, the constraint (7) becomes $c_2 = R_a x / (1-\gamma)$. Using equation (5) we obtain $c_2 = R_a(1-\gamma c_1)/(1-\gamma)$. After replacing c_2 by the previous expression in the objective function (4) and calculating the derivative relative to c_1 we obtain the first first-order condition $Eu'(c_1) = \delta R_a Eu'(c_2)$ which gives us equation (8) because c_1 and c_2 are deterministic, contrary to the case of autarky.

[10] $c_2^* \geq c_1^* \Leftrightarrow u'(c_2^*) \leq u'(c_1^*)$ since u' is a decreasing function. Using (8) we obtain $u'(c_2^*) \leq \delta R_a u'(c_2^*)$ or equivalently condition (10).

[11] In Allen and Gale (2000) the decentralization of the first best is realized through the interbank deposit market.

y^i represent respectively the amount invested in the investment project and the short asset of region *i* and z^i represents the investment in the investment project of region j≠ i. Therefore, given the distribution of states of the nature across the two regions (given by table 1), the portfolio (x^i, y^i, z^i) of bank *i=A,B* satisfies the following conditions:

$$x^i + y^i + z^i = 1 \quad (11)$$
$$y^i = \gamma c_1 \quad (12)$$
$$R_H x^i + R_L z^i = (1-\gamma)c_2 \quad (13)$$
$$R_L x^i + R_H z^i = (1-\gamma)c_2 \quad (14)$$

where condition (12) says that the liabilities of the bank at date 1 are covered by the short asset. Conditions (13) and (14) signify that the output of the long-term investment enables each bank to pay its depositors a constant amount c_2 whatever the state of the nature it takes place. Indeed, the liabilities of the bank at date 2 are covered by the sum of the output of the long asset across the two regions. It is simple to show[12] that each bank holds the same investment in the long-term projects of the two regions:

$$x^i = z^i = \frac{1-\gamma c_1}{2} \quad (15)$$

Hence by diversifying their long-term investment across the two regions, banks are able to satisfy their budget constraints in each state of the nature and at each date *t = 0, 1, 2* while providing their depositors with the optimal consumption allocation through a standard deposit contract.

2.5 Productivity Shock and Bankruptcy

We perturb the decentralized first best allocation by introducing a new state S_C where an unexpected shock affects the productivity of the investment projects in region A. Table 2 presents the characteristics of the different states of the economy in terms of the investment projects' return.

Hence in region *i=A,B* there is a probability of ½ that the state of the nature S_H occurs, a probability of ½ that S_L occurs and banks assign a zero probability to the state S_C at date 0. Thus, the sum of the probabilities of the states of nature equals 1. Since banks did not expect the realization of the state S_C, contracts and investment decisions at date *0* are the same as before. If S_1 or S_2 occurs, the first best allocation is realized. However, if the third state S_C occurs, the unexpected shock ε reduces the high return of the investment project in region A. We showed in section 2.4 that the diversification of the bank's assets across the two regions enables the decentralization of the optimal allocation. However, this strategy exposes the banks of region B to the negative unexpected shock happening in region A.

When state S_C occurs the average return of the investment projects across the economy is lower than expected. As illustrated in table 2, region B has the same return R_L but region A faces an unexpected productivity shock lowering the return R_H by ε. At date *1* all the depositors observe a signal (an economic indicator) revealing perfectly that the state S_C will occur at date *2*. Therefore, late depositors may withdraw their deposit prematurely at date 1 claiming they are early depositors and causing a bank-run in the process.

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[12] From (13) and (14) we have $R_H x^i + R_L z^i = R_L x^i + R_H z^i \Leftrightarrow (R_H - R_L)x^i = (R_H - R_L)z^i \Leftrightarrow x^i = z^i$

Definition 2. There is bankruptcy when the bank-run depletes all the assets of a bank at date 1 and the bank cannot honor its engagement as specified in the demand deposit contract.

The following proposition presents the condition of bankruptcy of banks in regions A and B.

Proposition 1.

If $\varepsilon \leq \varepsilon_m(r)$ then there is no bankruptcy.

If $\varepsilon > \varepsilon_m(r)$ then banks A and B are bankrupt.

with

$$\varepsilon_m(r) = 2\mathrm{R}_a - \frac{(1-\gamma)\,y}{(1-\mathrm{r}(1-\gamma))x^B}$$

(for the proof, see the appendix).

Figure 1 illustrates the results of proposition 1 in the diagram (r,ε) where r is the liquidity cost defined in section 2.1.

Region (i) represents the different combinations of (r,ε) for which neither bank A nor B are bankrupt. It is clear that for the bankruptcy to take place it is necessary that the productivity shock ε affecting the productivity of the long investment project in region A assets exceeds the minimum threshold $\varepsilon_m(r)$. Another interesting result to note is the fact that the economy becomes more vulnerable to the bankruptcy of banks A and B as the liquidation cost (1– r) decreases. This result is opposite to that obtained by Allen and Gale (2000) in the context of a liquidity shock that propagates through the interbank deposit market. It seems counterintuitive but in our case this is due to the fact that late consumers have lower incentives to withdraw their deposits early when the liquidation cost is higher. In Allen and Gale (2000) the liquidation of the long asset is not an option but always happens for the bank that faces the liquidity shock.

2.6 Partial Diversification and Bankruptcy of Bank B

Until now we have assumed complete symmetry between banks A and B. Let's now assume that bank B partially diversifies its portfolio and chooses the following portfolio $((1+\rho)x^B, y^B, (1-\rho)z^B)$ where y^B is defined by (12) and (x^B, z^B) are the optimal solution of the first best decentralization problem given by (15). This means that region B's bank over-invests in the long-term investment projects of its region and under-invest in the projects of region A. Naturally, this undermines the final remuneration of its late depositors who obtain $c_2^{'}$ verifying $c_1 < c_2^{'} \leq c_2$. Instead of equation (13) and (14) we have the following conditions for bank B:

$$R_H(1+\rho)x^B + R_L(1-\rho)z^B = (1-\gamma)c_2^{'} + \pi_\rho \quad (16)$$

$$R_L(1+\rho)x^B + R_H(1-\rho)z^B = (1-\gamma)c_2^{'} \quad (17)$$

where π_ρ represents an additional revenue for bank B which could be justified as a reserve requirement in the case of good performance of the region B. Although this portfolio allocation does not permit realizing the first-best allocation, it reduces the exposure of bank B to the unexpected productivity shock that takes place in the state S_C. The following proposition gives the new regions of bankruptcy across regions A and B.

Proposition 2.

Under the condition that the negative externality of bank A's bankruptcy is such that $\varphi > \varphi_m$, then bank B is less vulnerable to the bankruptcy triggered by the negative productivity shock relative to the case of full diversification. Indeed, the bankruptcy of bank B takes place if $\varepsilon > \varepsilon_m^B(\varphi, r) > \varepsilon_m(r)$ with

$$\varepsilon_m^B(\varphi, r) = 2\mathrm{R}_a + \frac{2\rho \mathrm{R}_L x^B - (1-\gamma)\, y}{(\varphi - \mathrm{r}(1-\gamma))x^B} \tag{18}$$

$$\varphi_m = 1 - \frac{2\rho \mathrm{R}_L x^B}{(1-\gamma)\, y}\big(1 - r(1-\gamma)\big) \tag{19}$$

(for the proof, see the appendix)

Definition 3. There is a financial contagion from bank A to bank B when the bankruptcy of the latter is due to the negative externality of the former's bankruptcy.

Lemma 2.

For a given (φ, r), financial contagion occurs when the productivity shock belongs to the region $\left[\varepsilon_m^B(\varphi, r), \varepsilon_m^B(1, r)\right]$.

Proof. Noting that by assumption $\varphi_m > r$ it is simple to show from (18) and (19) that $\partial \varepsilon_m^B(\varphi, r) / \partial r < 0$ and that $\partial \varepsilon_m^B(\varphi, r) / \partial \varphi > 0$. Figure 2 illustrates the results of proposition 2 in the diagram (r, ε) for two values $\varphi = 1$ and $\varphi < 1$. Compared to the results of proposition 1 (illustrated by figure 1) there is now two intermediate regions (iii) and (iv). In region (iii) the bankruptcy occurs only for bank A. In this region bank B is partially affected by the negative shock reducing the productivity of the long-term investment in region A as well as by the premature liquidation of the investment projects financed by bank A across the two regions. In region (iv) bank B is also bankrupted. However, this bankruptcy do not take place if there is no externality from the bankruptcy of bank A in the form of reduction of the long-term investment project return (i.e. if $\varphi = 1$). In other words, if bank A is (exogenously) rescued, bank B will not go bankrupt since the productivity shock in region (iv) is inferior to the threshold $\varepsilon_m^B(1, r)$. Therefore, we could qualify the region $\left[\varepsilon_m^B(\varphi, r), \varepsilon_m^B(1, r)\right]$ as the region of financial contagion which disappears if $\varphi = 1$.

3. The Presence of an Islamic Bank and Financial Contagion

Allen and Gale (2000) conclude that an interbank market for one-period loans opening at the second period could limit the contagion to other banks but cannot forestall the bankruptcy of the bank faced with a liquidity shock. In this section, we analyze if the presence of an IB in a third region C could prevent the bankruptcy of the CB in region B.

3.1 Characterization of the Islamic Bank

A representative IB is located in region C which contains a continuum [0,1] of consumers (depositors). Each consumer deposits his endowment of one unit of consumption good in one of the two deposit contracts offered by the IB: a *demand deposit contract* or a *profit sharing investment account*. I assume that a fraction β of the consumers hold a demand deposit contract enabling them to withdraw one unit of consumption at date 1 or at date 2 conditional on the liquidity shock they face at date 1. Hence, they do not share the bank's long asset risk but only want to keep their deposit intact in order to pay their expenditures. At date 1, only the fraction $\gamma\beta$ early consumers will withdraw their deposit and the IB will carry the remaining amount of deposit $(1-\gamma)\beta$ to date 2. The fraction $1-\beta$ of depositors hold a profit sharing investment account which enable them to withdraw their deposit only at date 2. The investment holders accept to be paid an amount contingent on the long asset return. For simplicity, we assume that the IB sector is competitive so that the share of the investment cash-flow that goes to the profit sharing investment account holders is $\mu = 1$ and the share of the IB is $1-\mu = 0$. Therefore, the payoff of the profit sharing investment account is given by

$$c_2^{is} = \begin{cases} R_H & \text{with a probability } 1/2 \\ R_L & \text{with a probability } 1/2 \end{cases} \tag{20}$$

3.2 The IB's Diversified Portfolio

We assume that the distribution of the long assets' returns in region C is ex-ante symmetrically correlated with those in regions A and B. Table 3 shows that we have the same distribution of returns for regions A and B as in table 2.

The only difference concerns the distribution of the return in region C where the IB exists, which is detailed in the following. There is an equal probability of ¼ for the high return and the low return to take place conditional on the realization of state S_1 or S_2. Therefore, the sum of the probabilities of all the possibilities for the IB is equal to 1. Let's also note that the IB, like the CBs, does not expect the realization of the state S_C which explains that the probability initially assigned to this state is zero. Let's now show that the IB could reduce the risk of the profit sharing investment account while holding the same expected return by diversifying its portfolio across the three regions by investing $(1-\beta)/2$ in region C, $(1-\beta)/4$ in region A and $(1-\beta)/4$ in region B. This strategy will provide the investment account holders with the following remuneration which replaces that presented in (20):

$$\hat{c}_2^{is} = \begin{cases} \frac{3}{4}R_H + \frac{1}{4}R_L & \text{with probability } \frac{1}{2} \\ \frac{3}{4}R_L + \frac{1}{4}R_H & \text{with probability } \frac{1}{2} \end{cases} \tag{21}$$

From (20) and (21) we obtain $E(\hat{c}_2^{is}) = E(c_2^{is})$ while $\text{var}(\hat{c}_2^{is}) < \text{var}(c_2^{is})$. This diversification strategy will however expose the IB to the negative effect of the unpredictable crisis state S_C. The following section explores the reaction of the IB in this state.

3.3. Could the Presence of an Islamic Bank reduce the Vulnerability to Financial Contagion?

When the investment account holders observe the negative shock affecting the assets of the IB they have no incentive to withdraw their investment early. This is not only because the contract stipulates that withdrawal is only possible at the maturity of the long asset, but also because (contrary to the conventional banks' late consumers) there is no additional benefit from doing this. Therefore, the negative shock on the asset side could be entirely passed-through to the liability side of the IB. However, fearing a confidence crisis that pushes its investment account holders to switch to other competing banks the IB may not remain passive particularly if bank A and bank B go bankrupt simultaneously, which results in the liquidation of a big proportion of the long assets in regions A and B. According to Syed Ali (2007) *Ihlas Finance House* allowed withdrawals from its Investment Accounts during the Turkish financial crisis of 2000-2001 to advertise its financial strength relative to its competitors or to evade the confidence crisis. This strategy which apparently was a strategic error that led to the closure of *Ihlas Finance House,* initially intended to keep the confidence of the clients. In our model, the IB adopts the strategy of the competitors of *Ihlas Finance House* who survived the crisis and refused to pay their investment account holders prematurely (Syed Ali 2007). In addition, we assumed that the Islamic banking sector is competitive in region C, thus our representative IB will try to reduce the pass-through of the CBs' bankruptcy to its investment account holders. This will be clarified in the rest of the section.

Assumption 1

The negative externality affecting the return of the long investment projects due to bankruptcy increases with the proportion l of liquidated investment project.

$$\partial(1-\varphi)/\partial l > 0 \tag{22}$$

Proposition 3

i) The presence of the IB in region C reduces the vulnerability of bank B to contagion.

ii) The higher the liquidity available for the IB the lower is the vulnerability of bank B to contagion.

Proof: If the crisis state S_C takes place the return of the investment account will be the following:

$$\hat{c}_2^{is,C} = \begin{cases} \frac{1}{4}R_{H,L} + \frac{1}{4}(R_H - \varepsilon) + \frac{1}{4}R_L & \text{if no bankruptcy} \\ \\ \frac{1}{4}R_{H,L} + \frac{1}{4}\varphi(R_H - \varepsilon) + \frac{1}{4}\varphi R_L & \text{if banks A and/or B is bankrupt} \end{cases} \tag{23}$$

The only solution for the IB in region C (to limit the deterioration of its assets) is to ensure the continuing financing of the maximum proportion of the long investment projects in region B. For this to happen the IB should be able to use its available liquidity at date 1 which is equal to $(1-\gamma)\beta$ (corresponding to the late demand deposits) to purchase the maximum proportion m of projects financed by bank A in region B. The IB should pay at least the unitary price rR_L which bank A could have otherwise obtained. The purchased asset will provide the IB with a payoff R_L generating an additional profit of $mR_L - mrR_L$. This operation will also reduce the negative externality affecting the return of the long investment projects which is captured through the new value of the parameter $\varphi' > \varphi$. Therefore, the IB will remunerate its investment account holders $\hat{\hat{c}}_2^{is,C} > \hat{c}_2^{is,C}$ given by

$$\hat{\hat{c}}_2^{is,C} = \left(\frac{1}{4}R_{H,L} + \frac{1}{4}\varphi'(R_H - \varepsilon) + \varphi'\frac{1}{4}R_L\right) + \frac{R_L(1-r)m}{1\text{-}\beta} \tag{24}$$

Using the results of lemma 2, it is clear that the region of bankruptcy of bank B is reduced by the above described behavior of the IB. Indeed, the latter by acquiring a proportion of the lon-term investment project of bank A is also reducing the exposure of bank B to the negative externality resulting from the bankruptcy of A.

Figure 3 illustrates the effect of the presence of the Islamic bank on the region of financial contagion which shrinks from $\left[\varepsilon_m^B(\varphi,r), \varepsilon_m^B(1,r)\right]$ to $\left[\varepsilon_m^B(\varphi',r), \varepsilon_m^B(1,r)\right]$.

Figure 3 shows that the region (iv) of contagion is now reduced compared to that in figure 2. The presence of the IB enlarged the region (iii) of non-bankruptcy of bank B in the case of bankruptcy of bank A. Hence, the IB's presence generates in this region the same effect on bank B as would a lender of last resort.

4. Conclusion

The share of Islamic banks in the banking system of many countries is growing. This transformation has generated new challenges for central banks with regards to their regulatory and supervisory frameworks. The transformation has also motivated many research studies to compare the behaviors of IBs to those of CBs. This paper sheds light on the optimal behavior of an IB when bankruptcy occurs in the conventional banking sector. To this end we develop a theoretical model inspired by Allen and Gale (2000). In the model I propose, an unexpected shock affects the banking assets in one region and generates financial contagion among the conventional banking sector. We show that the presence, in a third region, of an Islamic banking sector (offering demand deposit accounts as well as PLS investment accounts) reduces the vulnerability to financial contagion.

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Figure 1: The Bankruptcy Regions in the Diagram (r, ε)

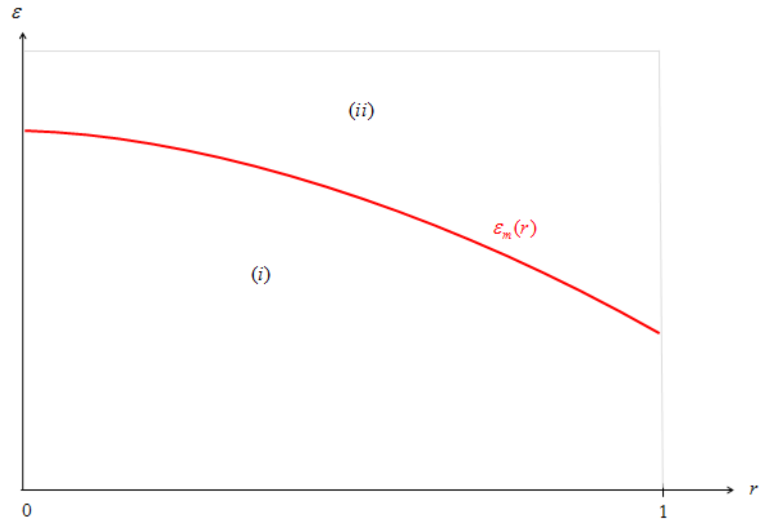

Figure 2: The Bankruptcy Regions in Case of Partial Diversification of Bank B

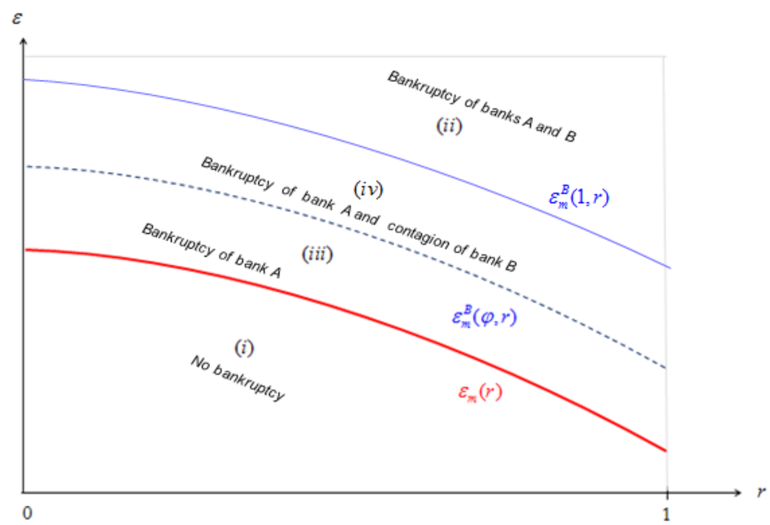

Figure 3: The Bankruptcy Regions in the diagram (φ, ε) in Presence of an Islamic Bank

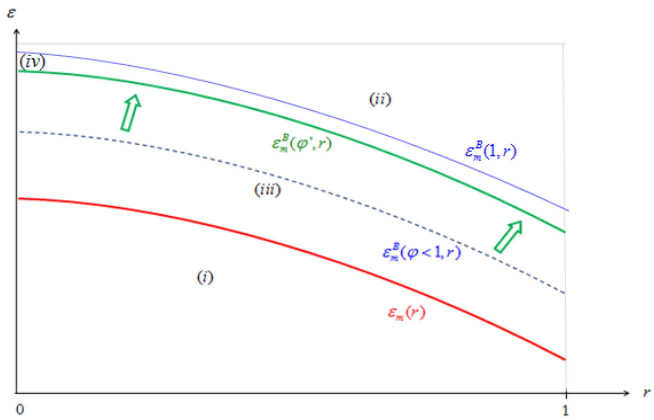

Table 1: Distribution of the State of Nature across the two Regions

	A	B	Probability
S_1	S_H	S_L	1/2
S_2	S_L	S_H	1/2

Table 2: Distribution of the Long Asset's Return with the Perturbation

	A	B	Probability
S_1	R_H	R_L	1/2
S_2	R_L	R_H	1/2
S_C	$R_H - \varepsilon$	R_L	0

Table 3: Distribution of the Long Asset's Return in the Presence of an IB

	A	B	*C*	*Probability*
S_1	R_H	R_L	R_H	1/4
			R_L	1/4
S_2	R_L	R_H	R_H	1/4
			R_L	1/4
S_C	$R_H - \varepsilon$	R_L	$R_{H,L}$	0

Appendix

Proof of Lemma 1.

Let's define a function f as following

$$f(y) = EU(y) = \gamma u(y/\gamma) + (1-\gamma)\delta u(R_a(1-y)/(1-\gamma)) \tag{A1}$$

The first derivative of f is given by

$$f'(y) = u'(y/\gamma) - \delta R_a u'\left[R_a(1-y)/(1-\gamma)\right] \tag{A2}$$

Given the conditions (8) and (9) we have

$$f'(y^*) = 0 \tag{A3}$$

It is easy to calculate the second derivative of f and to find that it is strictly negative given the assumption that the utility function u is strictly concave.

$$f''(y) = \frac{1}{\gamma}u''(y/\gamma) + \frac{\delta}{1-\gamma}u''(R_a(1-y)/(1-\gamma))$$

Therefore the function f is strictly concave with a maximum in y^*. Therefore we have

$$f(y) \le f(y^*) \text{ with equality } \Leftrightarrow y = y^* \tag{A4}$$

This is true in particular for $y = \gamma$

$$f(\gamma) = \gamma u(1) + (1-\gamma)\delta u(R_a) \le f(y^*) \tag{A5}$$

Let's now turn to the autarky allocation. From equations (3) we have the following relations

$$\begin{aligned} E(c_1^a) &= y^a + rR_a x^a = y^a + rR_a(1-y^a) < 1 \\ E(c_2^a) &= y^a + R_a x^a = (1-x^a) + R_a x^a < R_a \end{aligned} \tag{A6}$$

From equation (2) we obtain the following expression of the expected utility in autarky

$$EU^a = \gamma Eu(c_1^a) + (1-\gamma)\delta Eu(c_2^a) \tag{A7}$$

Since the utility function is strictly concave we have the following Jensen inequality $Eu(.) < uE(.)$ which enables us to obtain from (A7) and (A6)

$$EU^a < \gamma u\left[E(c_1^a)\right] + (1-\gamma)\delta u\left[E(c_2^a)\right] < \gamma u(1) + (1-\gamma)\delta u(R_a) \tag{A8}$$

Finally the proof is completed by combining (A8) and (A5) since we have

$$EU^a < \gamma u(1) + (1-\gamma)\delta u(R_a) \le f(y^*) = EU^* \tag{A9}$$

Where EU^* represents the expected utility of a consumer in the presence of the social planner.

Proof of Proposition 1

Late consumers of bank A have an incentive to withdraw their deposits prematurely at date 1 (and store it for consumption at date 2) if they obtain a larger payment than waiting until date 2. The value of bank A's total assets at date 2 is $(\mathrm{R_H} - \varepsilon)\mathrm{x^A} + \mathrm{R_L z^A}$ then late consumers will receive at date 2 a payment $\tilde{c}_{2A}$ given by:

$$\tilde{c}_{2A} = \frac{(\mathrm{R_H} - \varepsilon)\mathrm{x^A} + \mathrm{R_L z^A}}{1-\gamma} \tag{A10}$$

However, if they decide to withdraw their deposits at date 1, late consumers will trigger a bank run constraining bank A to liquidate its long asset and retrieve its investment in the long-term project of region B. They will receive at date 1 a payment $\hat{c}_{2A}$ given by:

$$\hat{c}_{2A} = y + r(\mathrm{R_H} - \varepsilon)\mathrm{x^A} + \mathrm{rR_L z^A} \tag{A11}$$

Since there is a symmetry of the problem relatively to banks A and B, the value of bank B total assets at date 2 is $\mathrm{R}_L \mathrm{x}^B + (\mathrm{R}_H \text{-} \varepsilon)\mathrm{z}^B$ and late consumers will receive at date 2 a payment $\tilde{c}_{2B}$ given by:

$$\tilde{c}_{2B} = \frac{\mathrm{R}_L \mathrm{x}^B + (\mathrm{R}_H \text{-} \varepsilon)\mathrm{z}^B}{1-\gamma} \tag{A12}$$

However, if they decide to withdraw their deposit at date t = 1 late consumers will trigger a bank run on B which liquidates its long asset and pays its depositors $\hat{c}_{2B}$ given by:

$$\hat{c}_{2B} = y + r\mathrm{R}_L \mathrm{x}^B + \mathrm{r}(\mathrm{R}_H \text{-} \varepsilon)\mathrm{z}^B \tag{A13}$$

Late consumers of bank i=A,B have an incentive to run on their bank if $\hat{c}_{2i} > \tilde{c}_{2i}$ which is equivalent after using equations (15), (A13) and (A14) to

$$\varepsilon > \varepsilon_m(r) = 2\mathrm{R}_a \text{-} \frac{(1-\gamma)y}{(1-\mathrm{r}(1-\gamma))x^B} \tag{A14}$$

Proof of Proposition2

Late consumers of bank B have an incentive to withdraw their deposit prematurely at date 1if they obtain a larger payment than waiting until date 2. The value of bank B's total assets at date 2 if bank A defaults ($\varepsilon > \varepsilon_m(r)$) is $\varphi R_L(1+\rho)x^B + \varphi(R_H - \varepsilon)(1-\rho)z^B$. Then late consumers will receive at date 2 a payment $\tilde{\tilde{c}}_{2B}$ given by:

$$\tilde{\tilde{c}}_{2B} = \frac{\varphi R_L(1+\rho)x^B + \varphi(R_H - \varepsilon)(1-\rho)z^B}{1-\gamma} \quad \text{(A15)}$$

However, if they decide to withdraw their deposit at date 1, late consumers will trigger a bank run constraining bank B to liquidate its long asset and retrieve its investment in the long-term project of region B. They will receive at date 1 a payment $\hat{\hat{c}}_{2B}$ given by:

$$\hat{\hat{c}}_{2B} = y + rR_L(1+\rho)x^B + r(R_H - \varepsilon)(1-\rho)z^B \quad \text{(A16)}$$

Late consumers of bank B have an incentive to run on their bank if $\hat{\hat{c}}_{2B} > \tilde{\tilde{c}}_{2B}$ which is equivalent after using equations (15), (A15) and (A16) to

$$\varepsilon > \varepsilon_m^B(\varphi, r) = 2\mathrm{R}_a + \frac{2\rho \mathrm{R}_L x^B - (1-\gamma)y}{(\varphi - \mathrm{r}(1-\gamma))x^B} \quad \text{(A17)}$$

Since we are in the case $\varepsilon > \varepsilon_m(r)$ then we should have the following condition on φ

$$\varphi > \varphi_m = 1 - \frac{2\rho \mathrm{R}_L x^B}{(1-\gamma)y}\left(1 - r(1-\gamma)\right)$$