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Send correspondence to:
Shawkat Hammoudeh
Lebow College of Business
Drexel University
hammousm@drexel.edu

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Abstract

This paper examines the dynamic relationship between global factors and herding behavior in the oil-rich frontier stock markets of the Gulf Cooperation Council (GCC), using a time-varying transition probability Markov-switching model (TVTP-MS). Our results suggest that the GCC frontier stock markets respond significantly to the global market conditions in two distinct ways: (i) global fundamentals and market factors play a significant role in determining volatility regimes in these frontier markets as well as their transitions from one regime to another; and (ii) shocks in global systematic financial risks significantly contribute to investor herding in these frontier markets. Higher levels of global risk indexes including the VIX and the FSI as well as positive changes in the U.S. stock market performance and in the price of oil govern the transitions out of low into higher volatility states during which herding behavior is found to be present. Finally, we discuss policy and portfolio diversification implications.

JEL Classifications: C32, G11, G15

Keywords: Herding; Gulf Arab stock markets; Dispersion shocks; Markov-switching; Time-varying probabilities.

ملخص

تبحث هذه الورقة العلاقة الديناميكية بين العوامل العالمية والسلوك الجمعى في أسواق الأسهم الغنية بالنفط في مجلس التعاون الخليجي (GCC) ، وذلك باستخدام نموذج متفاوتة الوقت واحتمال الانتقال ماركوف - التبديل (TVTP -MS) . نتائجنا تشير إلى أن أسواق الأسهم الخليجية تستجيب بشكل كبير لظروف السوق العالمية بطريقتين مختلفين: (1) الأساسيات العالمية و عوامل السوق تلعب دورا هاما في تحديد أنظمة التقلبات في هذه الأسواق الحدودية وكذلك انتقالهم من نظام واحد لآخر؛ و (2) منهجية الصدمات في المخاطر المالية العالمية تسهم إسهاما كبيرا في السلوك الجمعى للمستثمرين في هذه الأسواق. هناك ايضا مستويات أعلى من مؤشرات المخاطر العالمية بما في ذلك VIX و FSI فضلا عن التغيرات الإيجابية في الولايات المتحدة لأداء سوق الأسهم و أسعار النفط وتحكم التحولات من انخفاض في التقلبات فى الدول التي وجدت خلال السلوك الجمعى أن يكون حاضرا . وأخيرا ، نناقش سياسات وآثار تنويع المحفظة المالية.

1. Introduction

Investor behavior in emerging stock markets has been the topic of a number of studies in the literature. A growing strand of this literature focuses particularly on herding behavior in emerging and frontier stock markets (e.g. Tan et al. 2008; Chiang and Zheng 2010; Demirer et al. 2010). The literature in general provides inconclusive evidence on herding behavior in developed stock markets.[1] However, there seems to be a general consensus on the presence of such behavior in developing markets due to informational inefficiencies and other market specific factors (e.g. Demirer and Kutan 2006; Tan et al. 2008; Balcilar et al. 2013). On the other hand, there is nascent research on herding behavior among investors in the oil-exporting countries which have started to command more relevance in the world economy because of their possession of vast oil reserves and foreign assets (Demirer and Ulussever 2011). This nascent research however is largely limited to the domestic factors that affect investor behavior in these markets. Despite the evidence from the literature on international asset pricing that local factors are more significant determinants of asset returns compared to global factors (see for example Koedijk et al. 2002), in the case of the frontier Gulf Cooperation Council (GCC) markets, Hammoudeh and Li (2008) contend that GCC stock returns are affected more by global than domestic events. Therefore, it is yet to be explored if and how global factors influence investors' herding behavior in these frontier markets. Furthermore, considering the fact that GCC markets have varying degrees of openness available to foreign investors (e.g. Saudi Arabia is mostly inaccessible to foreign investors, whereas Dubai and Bahrain are relatively more open to foreign capital flows), the comparative analysis of these markets can provide valuable insight into the effects of financial globalization on developing stock markets.

To our knowledge, the only studies in the literature that address herding behavior in GCC markets are Demirer and Ulussever (2011) and Balcilar et al. (2013). Demirer and Ulussever (2011) focus solely on the effect of oil prices on herding but based on the static herding model, which does not address time-varying herding under different market regimes or structural breaks. On the other hand, Balcilar et al. (2013) follow a dynamic approach which takes into account herding under different market regimes, but without exploring how global factors might contribute to such behavior and govern the transition of volatility regimes in these frontier markets. This aspect of the relationship between global factors and return dynamics in these markets is especially of interest to policy makers in the region in order to devise strategies to manage the potential destabilizing effects of global shocks. Furthermore, the results should provide valuable insight into local and global investors regarding the financial integration of these markets with global markets, and thus can help with investment decisions, particularly given the reported potential diversification benefits of these markets for global investors (e.g. Yu and Hassan 2008; Cheng et al. 2010).

This study contributes to the literature in several aspects. First, we explore the role of global factors in driving herding behavior in frontier markets with a focus on the cash- and oil- rich GCC stock markets. More specifically, we examine the impacts of several global factors including the oil price, the U.S. market, risk indexes including the CBOE Volatility VIX index and St. Louis Federal Reserve's Financial Stress Index (FSI) and the dollar exchange rate on herding behavior. Therefore, this study expands the literature on emerging markets from a different perspective by exploring the effect of the global financial environment, which now exhibits greater instability and is surrounded by greater economic policy uncertainty since the

[1] Focusing on transaction data by institutional investors, a number of early papers including Lakonishok et al. (1992), Wermers (1999) and Jones et al. (1999) find a negligible level of herding behavior in the U.S. However, studies including Chang et al. (2000) and, more recently, Chiang and Zheng (2010) find evidence to the contrary in a number of advanced markets including the U.S.

onset of the 2008/2009 global financial crisis and the ensuing European sovereign debt crisis, on herding behavior among investors in frontier stock markets. Second, we propose a new regime-switching model where regime transitions are modeled in a time-varying framework as a function of global risk factors which might capture the different aspects of contagion in global markets. The herding tests, based on the time-varying transition probability Markov-switching (TVTP-MS) model for these markets, allow us to gain insight into the factors driving herding behavior from a unique perspective that has not been done in the literature. The TVTP-MS model not only accounts for direct effects of global market factors on herding behavior, but also provides insight into the effect of the global systematic financial risk factors on the transition probabilities between different market regimes where herding behavior may not or may strongly be present. By doing so, this study contributes both to the literature on herding and international asset pricing.

Our results suggest that the frontier stock markets in the GCC respond significantly to the global macroeconomic conditions in two distinct ways: (i) global fundamentals and market factors play a significant role in determining volatility regimes in these frontier markets as well as their transitions from one regime to another; and (ii) shocks in global systematic financial risks significantly contribute to investor herding in these frontier markets. Higher levels of risk indexes including the VIX and the FSI as well as positive changes in the U.S. market performance and in the price of oil are associated with transitions out of low into higher volatility market regimes during which herding behavior is observed. In short, the findings stress the significance of financial globalization on investor behavior even in frontier markets where policy makers allow limited access to foreign investors in order to protect these markets from the potentially destabilizing effects of globalization.

The remainder of the paper is organized as follows. Section 2 briefly summarizes the literature on the tests of herding behavior with a focus on emerging markets. Section 3 provides the description of the data and the testing methodology, while Section 4 explains the Markov-switching models to test the impacts of global market and financial risk effects on herding behavior in the GCC markets. Section 5 presents empirical results and Section 6 concludes the paper and discusses implications of the findings.

2. Previous Studies

A limited number of studies in the literature on emerging markets have focused on the stock markets in the Middle East and North Africa (MENA) region that have experienced high financial market returns despite the geopolitical instability surrounding this region. On the other hand, research on the oil-exporting GCC nations has been limited despite the growing presence of Gulf investors and sovereign wealth funds in global markets and the extraordinary performance of equity markets in the region fueled largely by soaring energy prices.

The early stream of the literature on the GCC stock markets focuses on return interdependence between these markets (Assaf 2003; Hammoudeh and Aleisa 2004; Hammoudeh and Li 2008, among others). This literature concentrates on the lead/lag relationships between the GCC member countries' stock market returns, while controlling mostly for the oil price. On the other hand, studies including Hammoudeh and Li (2008), Yu and Hassan (2008), Marashdeh et al. (2010), Ravichandran and Maloain (2010) and Cheng et al. (2010) focus on the interaction of these markets with global markets and document partial integration of these markets with international markets and the presence of diversification opportunities for global investors. However, this literature does not deal with the effect of global markets on structural breaks in equity returns and investor behavior in these frontier markets that are largely dominated by local, retail investors with limited financial environment.

The nascent strand of the literature that includes Balcilar et al. (2013) deals with uncertainty due to structural breaks, but does not adequately address the external factors which also affect herding behavior in those oil-rich, frontier markets as well as their transitions between market regimes over time. This paper contributes to the literature on developing markets by examining herding behavior in an environment characterized by structural breaks, external factor effects and time-varying probabilities governing regime transitions (see, for instance, Aloui and Jammazi 2009; Chen 2010; Dufrenot et al. 2011) .

3. Data and Testing Methodology

The literature offers several approaches to test the presence of investor herds in financial markets.[2] In this study, we follow a methodology originally proposed by Chang et al. (2000) and employed in a number of studies including Gleason at al. (2004) on exchange traded funds, Demirer and Kutan (2006) and Tan et al. (2008) on Chinese stocks, Demirer et al. (2010) on Taiwanese stocks and Chiang and Zheng (2010) on global stock markets, and more recently Balcilar et al. (2013) on GCC markets. The methodology builds the tests on the cross-sectional dispersion of individual stock returns around the market return and examines the patterns of return dispersions during periods of large market movements. Let $R_{i,t}$ be the return on stock *i* for period *t*. The dispersion of returns is measured by the cross-sectional average dispersion (CSAD) statistic defined as

$$CSAD_t = \frac{1}{n}\sum_{i=1}^{n}\left|R_{i,t} - R_{m,t}\right| \qquad (1)$$

where *n* is the number of stocks in the portfolio and $R_{m,t}$ is the return on the market portfolio. Following the rationale by Christie and Huang (1995) that herding behavior is more likely to occur during periods of market stress characterized by large price movements, the testing methodology examines the relationship between return dispersion and market return and estimates the following (domestic) static, linear model

$$CSAD_t = \alpha_0 + \alpha_1\left|R_{m,t}\right| + \alpha_2 R_{m,t}^2 + \varepsilon_t \qquad (2a)$$

where a significant and negative α_2 estimate is used as support for the presence of herding behavior. Note that the Capital Asset Pricing Model (CAPM) predicts an insignificant α_2 value as the firm level return is hypothesized to be linearly related to the market return.

The model in Equation (2a), however, is based on the local CAPM specification where a firm's return is explained by the domestic market factor only. However, as a number of studies in the international CAPM literature suggest, the domestic CAPM specification would be incorrect if there are additional risk factors perceived by local investors that might contribute to the return on a firm beyond what can be explained by the domestic market factor only (e.g. Stulz 1984/1995; Karolyi and Stulz 2003). This is simply because there might be greater (or less) global systematic risk in asset returns than is accounted for by the domestic market index. In fact, focusing on U.S.

[2] See, for example, Lakonishok et al. (1992), Christie and Huang (1995), Chang et al. (2000), and Hwang and Salmon (2004) for different methodologies offered in the literature. Demirer et al. (2010) provide a comparison of these methodologies.

firms, Francis et al. (2008) find that in addition to a premium for risk that is systematic with the local market, U.S. stocks have an economically significant premium for FX exposure.

In an ever integrated global financial system, as observed many times during the 1994 Mexican, 1997 Asian, and, more recently, the 2008 global market crises, contagion or transmission of shocks among global markets often drives investor behavior in local markets, stressing the importance of international determinants of stock returns in local markets, in particular in emerging and frontier markets with limited tools to manage market risks. One can certainly argue that among those markets are the GCC markets which have become more integrated with the global markets following the credit market crises (Khalifa et al. 2012). It is highly possible that herding in a certain frontier GCC market may be driven by global market shocks, transmitted either directly through international capital flows to or from these markets or through what is termed as the contagion of investor sentiments across stock markets. Furthermore, the GCC markets are also conventionally known to be highly sensitive to oil prices, since oil is a major source of revenue for their economies. In a related study, Khalifa et al. (2012) report that GCC stock markets are highly linked to the U.S. stock market. Similarly, Balcilar and Genc (2010) find that oil prices are informative in predicting the regime of the GCC stock markets.

Therefore, in order to avoid any misspecification errors regarding the pricing of GCC stocks, we extend the domestic linear model in Equation (2a) to a global factor model that includes the two significant determinants of global risk for GCC returns, i.e. the oil price and the S&P 500 index which represents a significant share of the global stock market activity, and estimate

$$CSAD_t = \alpha_0 + \alpha_1 \left| R_{m,t} \right| + \alpha_2 R_{m,t}^2 + \alpha_3 R_{US,t}^2 + \alpha_4 R_{O,t}^2 + \varepsilon_t \qquad (2b)$$

where $R_{US,t}$ and $R_{O,t}$ are the returns on the S&P 500 index and US price of WTI crude oil for period *t*, respectively. Note that if the domestic CAPM specification is valid, then α_3 and α_4 in Equation (2b) are expected to be insignificant, suggesting that the local market index indeed captures the global systematic risks in the cross-section of GCC returns. On the other hand, negative and significant values of α_3 and α_4 suggest that large changes in the global factors significantly contribute to herding behavior in these frontier markets.

For the empirical analysis, we use weekly closing prices for individual stocks listed on five GCC stock exchanges including those for Saudi Arabia, Dubai, Abu Dhabi, Kuwait, and Qatar obtained from Reuters, as well as for several exogenous global factors.[3] Following the literature on the financial integration of GCC markets to global markets, we include in the herding model the S&P 500 index and the WTI crude oil price, suggesting that these two factors capture global systematic risks that cannot be accounted for by the local market indices. However, as will be explained in more detail in Section 4.1, we also utilize a number of additional global factors in our analysis which may also have indirect effects on investor sentiment in these markets. These additional global factors are utilized in the estimation of regime transition probabilities since they may indirectly drive investor sentiment in the GCC markets and lead to market regime changes in these frontier markets.

The data covers the time series for each stock market until March 2012. Table 1 provides the summary statistics and the sample period for each GCC market as well as the global factors utilized. The data shows that all GCC markets have a positive average return during their sample

[3] Due to the differences in trading days between Western and GCC markets, weekly data is utilized.

periods, with Qatar having the highest return reflecting its growing oil and natural gas fortunes and Dubai having the lowest, possibly due to the recent real estate market crash. All GCC markets have an average return greater than that of the S&P 500 index, with the exception of Dubai. Similarly, these GCC markets are more volatile than the S&P 500 index, with the exception of Kuwait.

The return dispersion (CSAD) varies among the GCC markets, ranging from a low of 3.47% for Saudi Arabia to a high of 6.41% for Kuwait. The lowest cross-sectional dispersion is observed in Saudi Arabia and Qatar, possibly due to the government's use of their own domestic stock shares to stabilize their own markets (Balcilar et al. 2013; Hammoudeh and Aleisa 2004), which leads to a greater directional similarity in cross-sectional stock returns and thus lower return dispersions. Interestingly, we discern that Kuwait, which is the least volatile market in the GCC, experiences the highest return dispersion.

4. Testing Dynamic Effects of Global Factors on Herding Behavior

The domestic and international specifications in Equations (2a) and (2b), respectively, are static or linear in the sense that the model parameters are assumed to be constant over time, and thus fail to capture the impact of the latent variables on structural changes, leading to misspecification in these models. Dispersions are very noisy during periods of financial stress and are difficult to explain in linear regressions, even with very good regressors. Therefore, the herding tests based on Equations (2a) and (2b) fail to capture the dynamic nature of investor sentiment over the business cycle and different market phases when herding behavior may be present or otherwise.

One attractive specification that captures herding or non-herding over the different market phases in a consistent way with the return and volatility structure of markets is the Markov- switching (MS) model. Numerous studies, including Tyssedal and Tjostheim (1988), Hamilton (1988), Schwert (1989), Pagan and Schwert (1990), Sola and Timmermann (1994), Schaller and van Norden (1997), Kim et al. (1998), Kim and Nelson (1998), and Mayfield (1999), have utilized the conventional MS specification to model stock returns in different contexts other than herding. The MS models offer an advantage over the static or linear counterparts to model herding behavior due to their ability to reveal patterns beyond the traditional stylized facts, which only nonlinear models can generate. Several theoretical models that are consistent with regime-switching in stock returns including the rational stochastic bubble model of Blanchard and Watson (1982) and the switching fundamentals model based on the asset pricing model of Cecchetti et al. (1990).

In the case of herding tests, as noted by Christie and Huang (1995), herding relates to market volatility; and Bikhchandani and Sharma (2001) suggest that herding behavior is more likely to occur during periods of market stress. For this purpose, similar to Balcilar et al. (2013), we propose an MS specification that allows return dispersions to vary with market volatility in a regime-switching fashion. However, unlike Balcilar et al. (2013), we utilize a time-varying transition probability MS (TVTP-MS) model, which allows for the transitions among market regimes to be driven by global economic factors that may have significant impacts on investors' decisions. As explained earlier, these global factors are proxied by several macroeconomic variables, such as the exchange rate, oil price, and financial stress index. Furthermore, the TVTP-MS models allow the switches between herding, non-herding, and adverse herding to depend on these macroeconomic factors when regime switching is also a switch from one type of herding behavior to another. As will be discussed in the empirical results section, we find that regime switches are indeed simultaneous switches of volatility levels and herding behavior, supporting the description by Christie and Huang (1995) that links herding to volatility.

In order to capture the dynamic nature of the relationship between herding behavior and major global factors, we estimate the following 3-state Markov-switching model of the cross-sectional absolute dispersions, as warranted by the data[4].

$$CSAD_t = \alpha_{0,S_t} + \alpha_{1,S_t}\left|R_{m,t}\right| + \alpha_{2,S_t}R_{m,t}^2 + \alpha_{3,S_t}R_{US,t}^2 + \alpha_{4,S_t}R_{O,t}^2 + \varepsilon_t \quad (3)$$

where $\varepsilon_t \sim iid(0,\sigma_t^2)$ and S_t is a discrete regime variable taking values in {0,1,2} and following a three-state Markov process. The volatility term in Equation (3) is heteroscedastic with

$$\sigma_t^2 = \sigma_0^2 S_{0t} + \sigma_1^2 S_{1t} + \sigma_2^2 S_{2t} \quad (4)$$

where $S_{kt} = 1$ if $S_t = k$, and $S_{kt} = 0$, otherwise (k = 0, 1, 2). Equation (4) defines a regime dependent variance, which is equal to $\sigma_t^2 = \sigma_k^2$, k =0, 1, 2, for regimes 0, 1, and 2, respectively and allows the variance of the cross sectional dispersion of stock returns to switch across different regimes. Thus, the random variable S_t is defined as a three-state, first order Markov chain. The variance and parameters of Equation (3) switch across regimes, and the regime at any point in time follows a Markov chain whose realizations are unobservable.

As explained earlier, one of the novelties of this study is to estimate the global herding model in Equation (3) using time-varying transition probabilities across the different market regimes. A weakness of the standard MS models with constant transition probabilities is the implicit assumption that the expected durations of various regimes are constant over time, although they can vary across regimes. However, from a practical perspective, the length of time when herding behavior is present (or otherwise) in a market can fluctuate depending on the duration of the particular market volatility regime or the persistence of investor sentiment resulting from macroeconomic shocks. In order to resolve this weakness of the standard, constant probability MS specification, we assume that the transition probabilities related to the Markov chain in Equation (3) are time-varying and defined as $p_{ij,t} = P(S_t = i \mid S_{t-1} = j, \mathbf{Z}_{t-1})$ where $\mathbf{Z}_t$ is a vector of exogenous global variables possibly capturing investor sentiment, and thus driving herding behavior over regimes in these markets. Note that the variables in $\mathbf{Z}_t$ impact the transition probabilities with one lag since the transition probabilities governing the regime switches that occur from t-1 to t must be determined at time t-1. More specifically, let θ_{ij} be the vector of parameters of exogenous variables associated with the transition probability of switching from state j at time t-1 to state i at time t. The time-varying transition probabilities are then defined as

$$p_{ij,t} = \Phi(\mathbf{Z}_{ij,t-1}\theta_{ij}), \quad i = 0, 1 \text{ and } j = 0, 1, 2 \quad (5)$$

where $\Phi(.)$ is the normal cumulative distribution function (CDF). Thus, the specification in Equation (5) allows the transition probabilities across market regimes to be constant or time-varying. Another advantage of the TVTP specification in Equation (5) is that it restricts the

[4] We test for the optimal number of regimes and find that the data supports three regimes against a linear (1-regime) and the 2-regime alternatives. Several studies (Cakmakli et al. 2011; Guidolin and Timmermann 2006; Maheu et al. 2009) also find that that the 3-regime model better describes the stock return dynamics.

transition probabilities to values within the interval [0, 1], and also allows smooth adjustments.[5] Finally, the transition probabilities satisfy $\sum_{j=0}^{2} p_{ij,t} = 1$ for t=1, 2, …, T simultaneously.

Regarding the global factors utilized in the TVTP specification in Equation (5), in addition to the oil price and the U.S. market return discussed earlier, we utilize the three-month T-Bill rate since GCC countries officially or effectively peg their currencies to the U.S. dollar which might lead these markets to be somewhat sensitive to changes in the U.S. T-bill rate as their monetary policies are anchored to that of the United States. In addition, we use several exogenous measures of global risk represented by the dollar exchange rate index, the fear index (CBOE Volatility VIX index), and St. Louis Federal Reserve's financial stability index (FSI). VIX index is chosen as it captures the perception of volatility risk in the U.S. and global markets and the FSI index is chosen as it captures the impact of the 2007/2008 global financial crisis well.[6] The dollar index is also included in the analysis as the U.S. dollar is often regarded as the safe haven during times of market turbulence, and thus can reflect the level of global risk aversion. Thus, the vector **Z** = [z_i] (i=0,1,…,6) in Equation (5) is defined as **Z** = (1, Dollar Index Return, VIX, S&P 500 Return, WTI Return, FSI, TB3). For instance, according to the specification in Equation (5), the parameter, $\theta_{12,4}$, captures the impact of oil price changes (z_{4t}) on the transition probability from market regime 2 to regime 1, i.e. p_{12}.

Overall, the global TVTP-MS model specified in Equations (3) through (5) allows us to gain insight into the factors driving herding behavior from a unique perspective that has not been done in the literature. The model not only accounts for direct effects of global factors on the dispersion of stock returns consistent with the international CAPM specification for GCC returns, but also provides insight into the effect of the global factors, including risk indexes that relate to investor sentiment and contagion, on the transitions between different market regimes.

5. Empirical Results

In this section, we present our analysis of the static global herding model of Equation (2b) and the TVTP-MS model described in Equations (3) through (5). As will be discussed later in this section, the static models given in Equations (2a) and (2b) have been rejected by both the standard and Davies (1987) LR tests against the MS alternatives. Furthermore, our tests indicate that the constant transition probability MS model is rejected against the time-varying transition probability specification, suggesting that the TVTP-MS model provides the best explanatory power as will be discussed later.

5.1 The results of the static model

Table 2 presents our estimates for the static global herding model described in Equation (2b).[7] The static models yield evidence consistent with herding behavior for Qatar and Saudi Arabia only. Although, the herding coefficients, α_2, are negative for all five markets, they are found to be insignificant for Abu Dhabi, Dubai and Kuwait. The Qatari economy depends heavily on the hydrocarbon markets which are linked to the performance of major global economies, while the

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[5] Smoothed probabilities are calculated following Kim (1994). Further detail about the restrictions and computation of the time-varying transition probabilities is provided in the Appendix.

[6] The FSI is the primary principal component of 18 financial variables related to interest rates, yield spreads, and other indicators. http://research.stlouisfed.org/publications/net/NETJan2010Appendix.pdf

[7] The results for the local herding model given in Equation (2a) are somewhat similar to those of the global model of Equation (2b) and are available upon request.

Saudi market suffers from the lack of breadth and depth relative to the size of the petrodollars the country receives, and is considered to be the most strict market in the GCC region in terms of access to foreign investments.

In the case of global factors, we find that both Qatar and Saudi Arabia herd around the U.S. market, whereas no significance of the oil price is observed. Note that many Saudi investors "park" their substantial savings in the United States for safety and hedging purposes which might explain the significance of the U.S. market factor in Saudi Arabia. The insignificant findings for the global factors in the rest of the GCC markets are most likely due to regime dependence in herding behavior which the static model fails to capture, as the regime-based models will indicate later.

5.2 The regime-switching model

5.2.1 Model selection and estimation procedure

The empirical procedure for building MS models suitable for our case starts with identifying a possible set of models to consider. The models vary in terms of the number of regimes (*k*) and the specification of variance. Additionally, we estimate MS models with constant transition probabilities, i.e., $p_{ij,t} = p_{ij}$, and test variants of these models against the variants of the TVTP-MS model. Once a specific MS model is estimated, the next step is to test for the presence of nonlinearities in the data. It is of interest to test whether nonlinearity adds any explanatory power to the static, constant coefficient model in Equation (2b). When testing the MS model against the static alternative, or a *k* regime model against a (*k*-1) regime model, the transition probabilities are not identified under the null and, therefore, the standard distribution theory does not apply. For this purpose, we employed two separate sets of tests based on the chi-square statistic suggested by Ang and Bekaert (2002) and the LR statistic as derived by Davies (1987). For space considerations, we do not report the full set of model selection tests, however the detailed tables are available upon request.

Once nonlinearity is established, we choose the number of regimes and the type of the MS model based on both the likelihood-ratio statistic of Davies (1987) and the Akaike information Criterion (AIC). Krolzig (1997) and Psaradakis and Spagnolo (2003) suggest selecting the number of regimes and the type of the MS model using the AIC criterion and, based on Monte Carlo experiments, Psaradakis and Spagnolo (2003) show that the AIC is generally successful in selecting the correct model. The comparisons of the log likelihoods and the AICs for the static global model in Equation (2b) and variants of regime-dependent herding models, i.e., the 2- and 3-regime heteroscedastic MS variants and the homoscedastic alternatives, yield evidence in favor of the 3-state TVTP-MS specification for all GCC markets. In order to further check the robustness of the three-regime specifications, we include in the model several combinations of dummies that correspond to spikes in the CSAD values exceeding three standard deviations of the mean, with the restriction that no more than 18 dummies will be included in any case.[8] None of the combinations of the dummies changes the three-regime TVTP-MSH results. In fact, the inclusion of the dummies even enhances the test results in favor of the three market regimes in some cases. Thus, we conclude that the three-regime specification for the GCC markets is not spurious and corresponds to true regimes. Once again, the results of the tests to identify the best model for each country are not included for space considerations and are available upon request.

[8] It is possible that the third regime is spurious and corresponds to few spikes in the data. Nielsen and Olesen (2001) find that the third regime for Danish stock market is a figment of the data which disappears when dummy variables are included corresponding to few spikes in the data. But this is not the case in our MS models.

The parameters of the TVTP-MS herding model described in Equations (3) through (5) are estimated using the maximum likelihood method, assuming that the errors are normally distributed. However, since the regime probabilities are not known, they are evaluated using the filtering procedure of Hamilton (1990) followed by the smoothing algorithm of Kim (1994). Once the regime probabilities are evaluated, the log-likelihood of the TVTP-MS model is maximized using the trust-region-dogleg algorithm as a function of the parameters in Equation (3), the regime-dependent variance defined in Equation (4), and the parameters of the TVTP equation given in Equation (5). The estimates are obtained by maximizing the log-likelihood subject to the constraint that the probabilities lie between 0 and 1 and sum to unity.

5.2.2 Results of the TVTP-MS model

Table 3 presents our findings for the global TVTP-MS model specified in Equations (3) through (5). The estimates for the volatility terms (σ_k^2, k=0, 1, 2) for each state clearly differentiate each regime in terms of the level of market volatility. In the case of Saudi Arabia, for example, the estimated volatility (standard deviation) value of 0.0259 in regime 2 (crash regime). This is almost four times as high as the volatility estimate of 0.007 for regime 0 (low volatility regime), clearly suggesting the presence of more than one market regime, each identified with a different level of volatility. Regarding the herding tests, we find significant evidence consistent with the presence of herding behavior in all GCC markets during the crash regime, where herding is also observed in Abu Dhabi and Kuwait during the high volatility regime (i.e. regime 1). It is worth emphasizing that the static herding model specified in Equation (2b) fails to detect herding for Abu Dhabi, Dubai, and Kuwait when in fact herding occurs in these markets during periods of high volatility. These findings are consistent with earlier studies including Christie and Huang (1995), Chang et al. (2000) and Bikhchandani and Sharma (2001), suggesting that investors will be more likely to suppress their own beliefs and copy the behavior of others during periods of market stress. They are also consistent with the rationale behind the testing methodology which is based on the relationship between return dispersions and market returns during periods of market stress.

Consistent with the literature on the international CAPM, all of the coefficients representing the global factors (i.e. $\alpha_{3,k}$ and $\alpha_{4,k}$, k=0,1,2) in Equation (3) are found to be highly significant, implying the presence of global systematic risks priced into the cross-section of GCC stock returns. With regard to the impact of the global factors, we observe the strongest results for Abu Dhabi, Kuwait, Qatar and Saudi Arabia where both the U.S. stock market and the oil price are found to significantly contribute to herding behavior in these markets during the market regimes where herding is found to be present in each country. These four markets are associated with major oil-exporters and prominent members of OPEC and have the closest relation with the oil market. On the other hand, the U.S. stock market, but not the oil price, is found to significantly contribute to herding in Dubai during the crash volatility regime. Interestingly, Dubai is the only the GCC market where the oil price is found to be insignificant in driving herding behavior which can be explained by its relatively lesser and decreasing dependence on oil exports and its greater diversification in the real estate and tourism sectors. On the other hand, it is also interesting that shocks in the U.S. equity market significantly contribute to herding behavior across all GCC markets during periods of market turbulence, further stressing the financial integration of these markets with global stock markets regardless of the level of access to foreign investors. However, the finding of such global effects during periods of market stress only suggests that financial integration is most likely due to contagion, which occurs through international capital flows and macroeconomic linkages between these countries and western markets. This finding is also consistent with Khalifa et al. (2012) who, using a different methodology, find evidence of volatility spillover from the US stock market and the oil market to GCC stock markets under two regimes.

5.2.3 Persistence of market regimes

It may be helpful to examine the expected duration of regimes in order to make inferences about the persistence of market regimes. However, since transition probabilities are time-varying, thereby leading the regime durations to vary over time, we consider the average duration measures, $\tau_k, k = 0,1,2$, reported in Table 3. We observe that the low volatility regime (regime 0) has the greatest duration for all five GCC markets. The average duration of the crash volatility regime (regime 2) is generally higher than that of the high volatility regime in all markets except for Abu Dhabi and Dubai. The longest average durations for the crash regime are observed in Dubai and Kuwait (7.98 and 7.86 weeks, respectively) whereas the average duration of the crash regime for Abu Dhabi, Saudi Arabia, and Qatar are much lower with 2.27, 2.60 and 3.86 weeks, respectively, suggesting that the crash regime is quite persistent for Dubai and Kuwait with an average duration of nearly two months. These two countries are known for historical real estate and stock market crashes.

Figures 1-5 plot the smoothed probability estimates, which allow us to examine the dynamic evolution of the transition probabilities and market regimes over time. The first panel of each figure additionally plots the corresponding returns. The smoothed probability estimates in Figures 1-5 indicate a clear and sharp classification structure, switches are mostly instantaneous and the uncertainty about the active regime is very low. This is particularly clear for Dubai and Kuwait, but also holds for the other three markets. In general, the order of the regime changes that is indicated by the gray areas in the figures can be summarized as follows: the low volatility regime is followed by the crash regime, and the high volatility follows the crash (LCH). For instance, we find that Abu Dhabi is in the low volatility regime until the end of 2007, then a crash regime becomes active at the end of 2007 and early 2008, which is followed by the high volatility regime. In the case of Dubai, its stock market stays in the low volatility regime until late 2008, then moves to the crash volatility for a prolonged period until almost mid-2009, and finally moves to the high volatility regime. The extreme or crash volatility periods during 2007-2009 are mostly followed by periods of high volatility. The LCH order of the regimes in these GCC markets is noteworthy for most periods and should be a cause of concern for investors.

Panels (a) in Figures 1 through 5 display returns, along with periods where herding is detected. In Figure 1(a), for example, we see that herding was a persistent phenomenon in Saudi Arabia from 2006 to mid-2007 and from the end of mid-2008 to mid-2009. This period certainly witnessed several crashes and extreme volatilities. Abu Dhabi, Kuwait, and Qatar are the three markets where herding is detected frequently and over long periods, as shown in Figures 1(a), 3(a), and 4(a), respectively. The stock market in Abu Dhabi displays herding almost in all periods since 2001. The support for herding is stronger and more persistent for the most volatile periods. Overall, among the five GCC markets, Abu Dhabi, Kuwait, and Qatar are unique with strong and persistent herding almost in all periods. Herding is less frequent for Dubai and Saudi Arabia, but occurs persistently for some periods during 2004-2009.

5.2.4 Time-varying transition probabilities

The most important feature of the TVTP estimates ($p_{ij,t}$) given in Figures 6-10 is the highly time-varying nature of the transition probabilities, particularly in the case of cross-regime switches, i.e. $p_{ij,t}$ for $i \neq j$. The variations in transition probabilities, due to macroeconomic factors as specified in Equation (5), are mostly swift, reflecting the clear separation in regime classifications. The transition probability estimates for $p_{00,t}$ indicate that the low volatility regime is highly persistent and displays the smoothest change over time, particularly for Dubai, Kuwait, and Saudi Arabia; and for Abu Dhabi and Qatar to a lesser extent. The $p_{00,t}$ estimates for Dubai, Kuwait, and Saudi Arabia are highly persistent, while for Abu Dhabi and Qatar, the persistence is less than moderate. These features are also reflected in the average duration estimates for the low volatility regime.

The transition probability estimates, $p_{02,t}$, for switching from the crash regime to the low volatility regime are essentially zero for Abu Dhabi, Kuwait, Qatar, and Saudi Arabia, except a few spikes reaching 0.50 to 0.83. But for the crisis-ridden Dubai, however, $p_{02,t}$ stays at around 0.92 between the years 2006 and 2009. This finding suggests that these markets do not switch directly from crash to tranquility regimes, and that the high volatility regime follows crashes, creating volatility clustering following the crashes. This is supported by very high and somewhat persistent transition probability estimates for switching from the crash to the high volatility regime, $p_{12,t}$, particularly for Abu Dhabi, Qatar, and Saudi Arabia. Probably the most important feature of the transition probability estimates is the very low probability of moving from the crash regime to the low volatility and from the low to the high volatility. The transition probability estimates for switching from the low to the high volatility, $p_{10,t}$, are essentially zero for most periods for Abu Dhabi, Dubai, Kuwait, and Saudi Arabia and are only at high levels before 2007 for Qatar. This implies that a crash is mostly a necessary regime to move from low to high volatility for these markets, particularly after 2006. This is reinforced by the high transition probability estimates, which reach 60% in 2005 (Qatar) and 95% in 2008 (Dubai), of switching from the low volatility regime in one period to the crash regime in the next. The probability of switching from the low volatility to the crash regimes $p_{20,t}$ rises from zero to significant levels after 2005 in all GCC markets.

Overall, these empirical results regarding the transition probabilities suggest that the crash regime is the intermediate regime between the low volatility and the high volatility regimes. It is important to note that the regime transition order is different from the transition structure for developed markets which have the common order of "low, high, crash volatility' that provides investors with a "warning signal" of a looming crash expected to follow periods of increased volatility. On the other hand, as the findings suggest, a high volatility regime occurring right before the crash is rarely observed in the GCC markets. That is, crashes are swift and the occurrence of heightened volatility right before crashes is not commonly observed. However, the crash periods are associated with 'extreme' volatility and the periods following the crashes are characterized by high volatility in the GCC markets, leading to volatility clustering. Therefore, crashes are hard to predict from volatility changes in the GCC markets. The crash regime is highly persistent for Dubai, Kuwait, Qatar; and is as persistent as the high volatility regime for Saudi Arabia.

5.2.5 Global factors and regime transition probabilities

As explained in Section 4, the parameters θ_{ij}, $i = 0,1$ and $j = 0,1,2$ in Equation (5) capture the dynamic effects of global factors on transition probabilities across regimes. Significant parameter estimates imply that these factors play a role in leading GCC markets from one regime to another, possibly driving herding regimes. As discussed earlier, we consider the global variables including oil price changes, the S&P 500 index returns, the VIX volatility index, the Financial Stress Index (*FSI*), the change in the dollar exchange rate index, and the U.S 3-month Treasury bill rate (*TB3*). Thus, the l^{th} element of the vector $\hat{\theta}_{ij}$, that is $\hat{\theta}_{ij,l}$ for $i = 0,1$ and $j = 0,1,2$ is defined as {l = 0 (constant), 1 (dollar index returns), 2 (VIX), 3 (S&P 500 returns), 4 (WTI returns), 5 (*FSI*), 6 (*TB3*)} with six parameter estimates for each variable since there are only six free transition probabilities.

The most notable feature of the estimates given in Table 3 is the significance of almost all parameters in all of the TVTP equations for $p_{ij,t}$. The only exceptions are the insignificant parameter estimates for $p_{02,t}$ and several for $p_{12,t}$ which are associated with the S&P 500 index and T-bill rate, implying that the U.S. market returns and T-bill rate do not have a significant impact when GCC markets are moving from the crash regime to both the low and high volatility regimes, suggesting the dominance of extreme volatility over shocks from those U.S. factors. A second noteworthy feature is the observed parameter heterogeneity across the GCC markets' transition probabilities in terms of the signs of the estimates, suggesting that the global factors do not influence transition probabilities across the markets in a uniform way. An important exception is the oil return which is found to have a positive impact on $p_{00,t}$, suggesting that bullish oil prices increase the probability that GCC markets stay in the low volatility regime. On the other hand, we find that the higher the return on the U.S. market, the lower the probability that GCC markets stay in the low probability regime with the exception of Abu Dhabi. This suggests that the bull market conditions in the U.S. makes GCC markets more likely to switch out of the low volatility regime.

In the case of the risk indexes, we find that an increase in the fear index (VIX) generally lowers the probability that GCC markets will stay in the low volatility regime in the next period when they are in the low volatility regime in the current period. Similarly, the financial stability index (*FSI*) is also found to have a consistent effect on the regime transitions in the GCC stock markets. For instance, an increase in the *FSI* index increases the probability that the GCC markets will move from the low to the high volatility regimes, with the exception of Dubai. These findings suggest that fear and market stress in the U.S., proxied by the VIX and the FSI, are indeed picked up in frontier markets.

Overall, the evidence indicates that the frontier stock markets in the GCC respond significantly to the global macroeconomic conditions in two distinct ways. (i) The global fundamentals and market factors including the U.S. market performance and the price of oil *directly* drive herding behavior in the GCC; and (ii) global market and systematic financial risk factors also play a significant role in determining volatility regimes of the GCC markets as well as their transitions from one regime to another. In short, the global factors are found to be driving forces for herding behavior in the GCC markets through their impacts on investor sentiments.

6. Implications and Conclusions

This paper examines the dynamic relationship between global fundamentals and market factors and herding behavior in the five oil rich, frontier stock markets of the Gulf Cooperation Council (Abu Dubai, Dubai, Kuwait, Qatar and Saud Arabia), using a time-varying transition probability Markov-switching model (TVTP-MS). We find evidence of herding behavior in all five GCC stock markets during the crash regime, particularly, frequently and over long periods for Abu Dhabi, Kuwait and Qatar. The average duration of the crash regime is quite long and comparable to that of the high volatility regime. The striking result about volatility persistence is the exceptionally high regime duration for Dubai and Kuwait in both the low and the crash regimes relative to the other GCC markets. Kuwait is known for its stock market crashes and Dubai recently had a debilitating crash. Volatility in the low volatility regime may persist for more than 20 weeks for Dubai and 34 weeks for Kuwait. In the crash regime, this volatility could also persist for nearly two months. These findings imply that equity investment, particularly in those two markets, goes through tumultuous rides. Policy makers in Dubai and Kuwait should be cognizant of these findings, and thereby build safety circuits and hedging instruments to deal with volatility persistence in those two markets. It will be a remiss if we do not point out that the volatility in Abu Dhabi is anti-persistent even in the low volatility environment. This speaks out for the different type of spending and economic growth policies pursued in this oil-rich emirate, which has the second largest sovereign wealth fund in the world. It is safe to say that the Abu Dhabi market is for the risk-averse and faint-hearted investors.

Examining the smoothed probability estimates, we conclude that the order of the regimes for the GCC markets is generally: the low volatility is followed by the crash, and the high volatility follows the crash (LCH). This comes in contrast to the order of the major equity markets which follow the order LHC. The GCC markets' LCH order implies that the GCC markets, in addition to being persistently volatile, do not have an adequate warning signal of when the crash encroaches on those markets. This is alarming when combined with inadequate tools to hedge market risks.

In the case of global effects, our results suggest that the frontier stock markets in the GCC respond significantly to the global macroeconomic conditions in two distinct ways. (i) The global fundamentals and market factors including the U.S. market performance and the price of oil *directly* drive herding behavior in the GCC; and (ii) global financial risk factors also play a significant role in determining volatility regimes of the GCC markets as well as their transitions from one regime to another. Higher levels of risk indexes including the VIX and the FSI as well as positive changes in the U.S. market performance and in the price of oil are associated with transitions out of low into higher volatility states during which herding behavior is observed. Interestingly, this evidence comes despite the fact that most GCC markets protect themselves from foreign investors by putting up barriers to entry, partly in the hope of reducing the impact of global volatilities on their markets. The results on the impact of the global shocks on their markets are still significant, pointing to the strong integration of these frontier markets with the world's global markets. All in all, these results confirm the previous ones stated earlier that the GCC markets are well integrated with other financial and oil markets. If the objective behind setting up barriers to entry of foreign investors is to reduce market panic, herding, persistence of volatility and switching between volatility regimes, this study concludes the other way.

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Figure 1: Return and Smoothed Probability of 3-Regime Nonlinear TVTP-MS Model for Abu Dhabi Stock Market

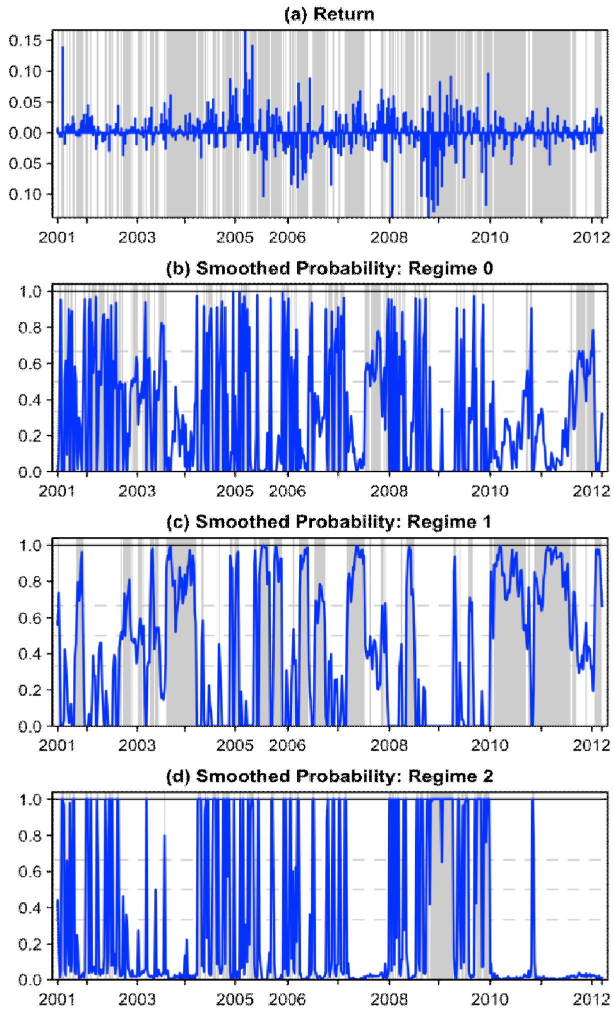


Figure 2: Return and Smoothed Probability of 3-Regime Nonlinear TVTP-MS Model for Dubai Stock Market

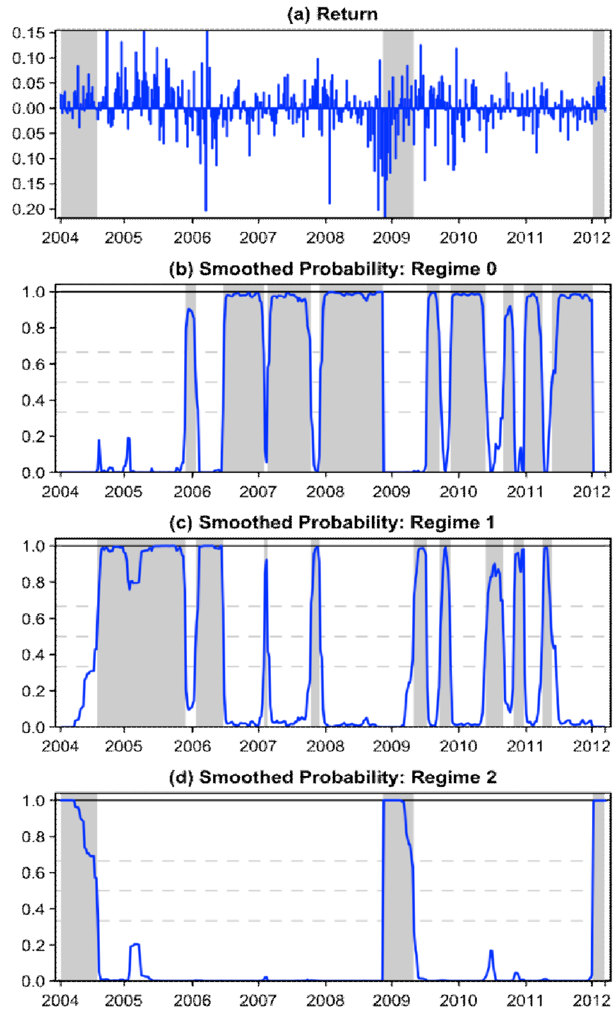
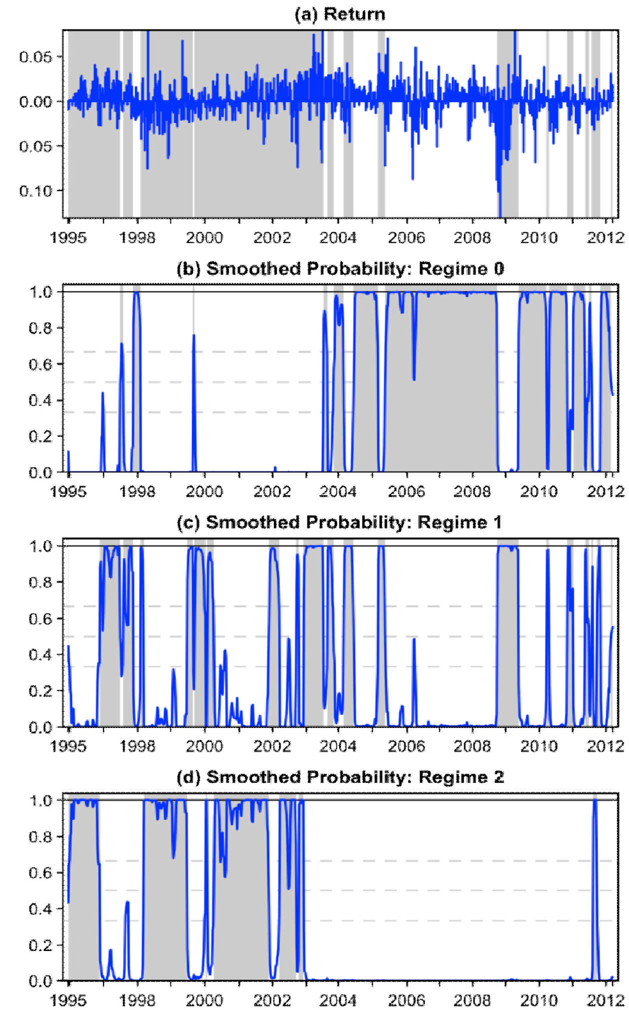


Figure 3: Return and Smoothed Probability of 3-Regime Nonlinear TVTP-MS Model for Kuwait Stock Market



Note: Figure (a) plots the market return. The shaded regions in Figure (a) correspond to regimes where herding is supported with negative coefficients on squared returns in Equation (3). Figures (b)-(d) plot the smoothed regime probabilities for the 3-regime nonlinear TVTP-MS model in Equations (3) through (5). The shaded regions in Figures (b)-(d) correspond to the maximum smoothed probability among the three smoothed probabilities. Regime 0 is the low volatility, Regime 1 is the high volatility and Regime 2 is the crash or extreme volatility.

Figure 4: Return and Smoothed Probability of 3-Regime Nonlinear TVTP-MS Model for Qatar Stock Market

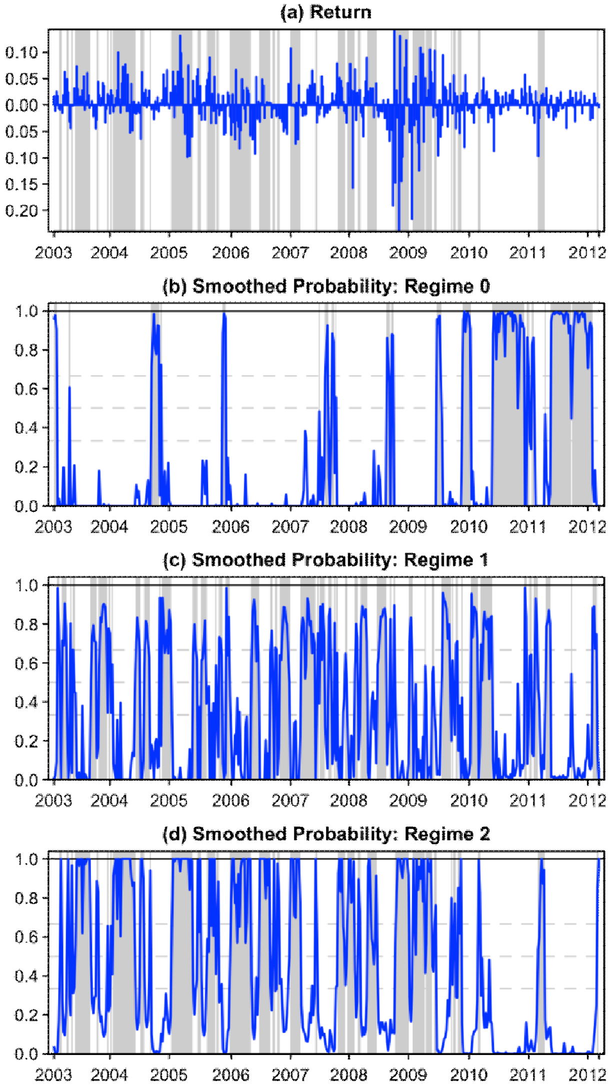
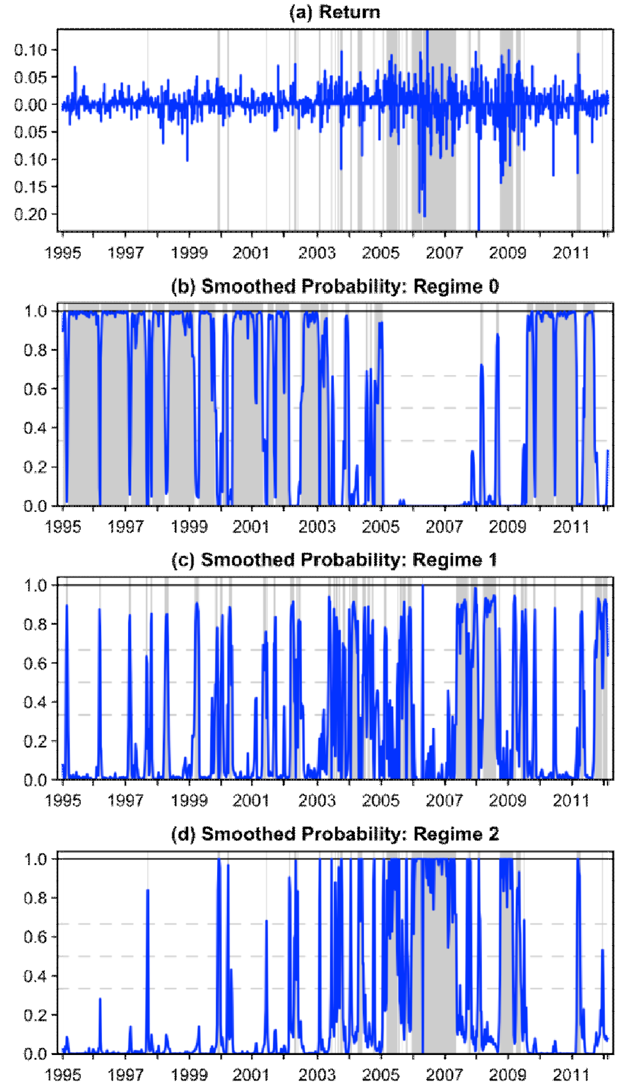


Figure 5: Return and Smoothed Probability of 3-Regime Nonlinear TVTP-MS Model for Saudi Arabia Stock Market



Notes: See Figure 1

Figure 6: TVTP Estimates for Abu Dhabi[9]

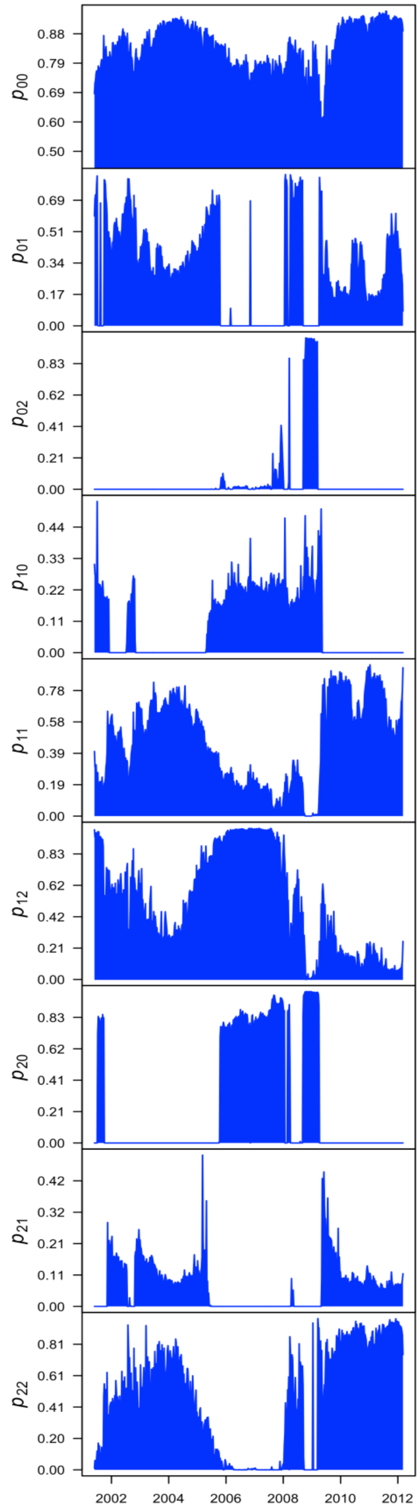


Figure 7: TVTP Estimates for Dubai

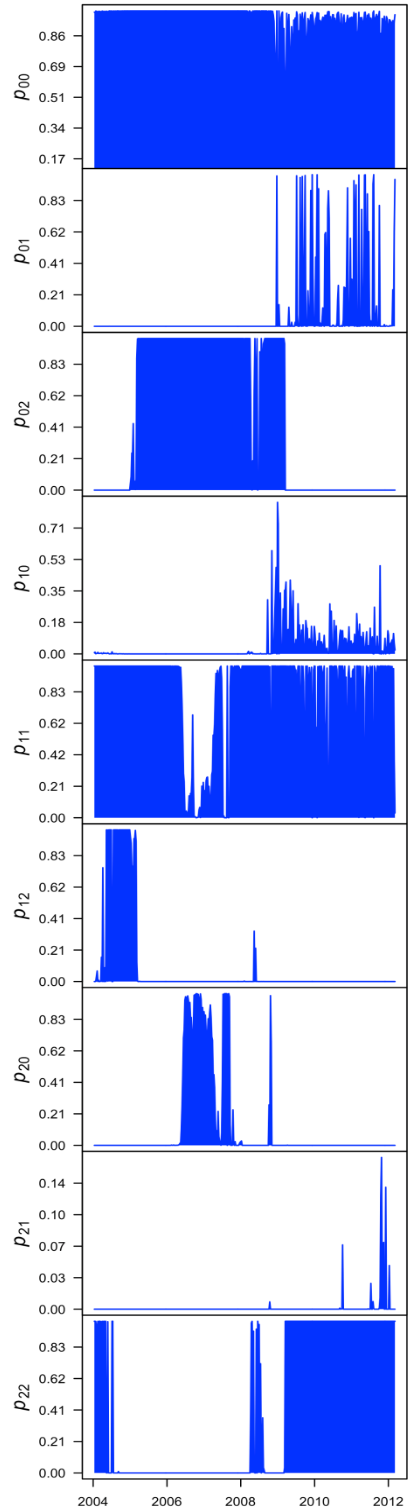
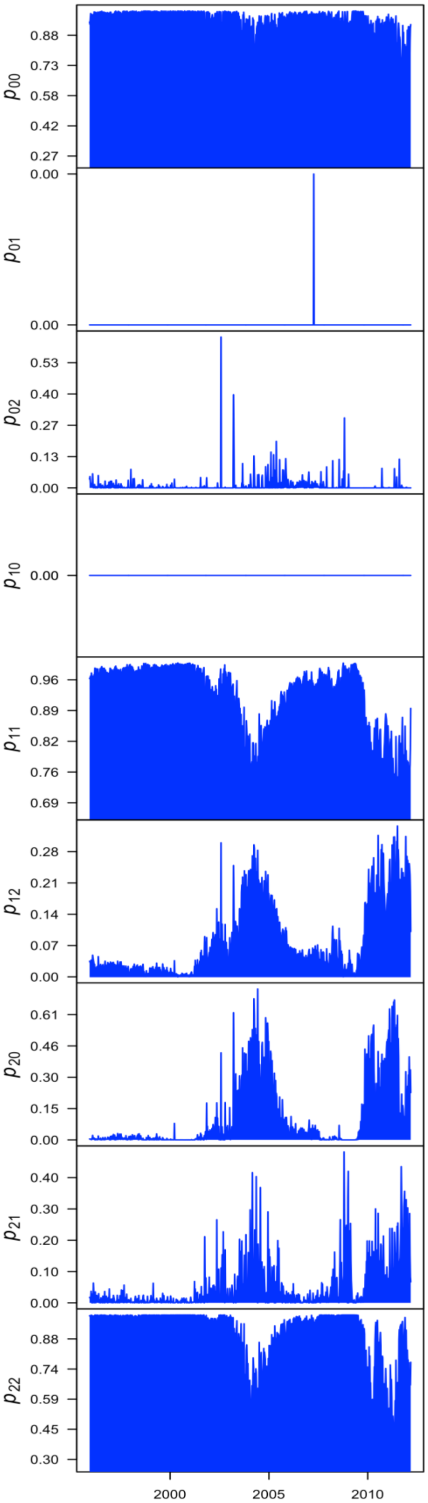


Figure 8: TVTP Estimates for Kuwait



[9] Note: Figure plots the estimates of time varying transition probabilities $p_{ij,t+1}$ = P(S_{t+1}=i|S_t=j), the transition probability from state j to state i, defined in Equation (6).

Figure 9: TVTP Estimates for Qatar

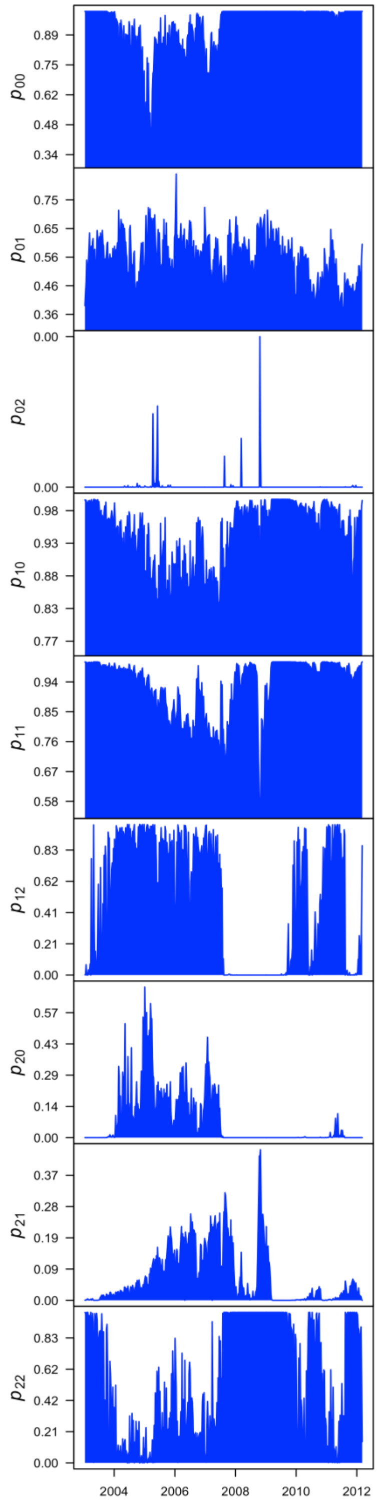
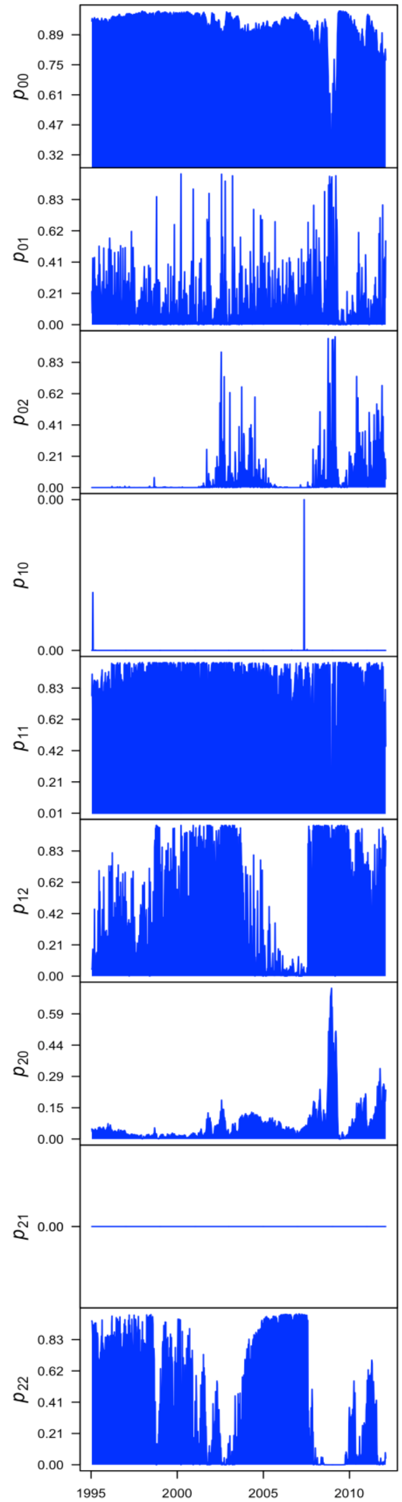


Figure 10: TVTP Estimates for Saudi Arabia



Note: See Figure 6.

Table 1: Descriptive Statistics

	Mean	S.D.	Min	Max	*n*	Sample Period
Abu Dhabi						5/29/2001-3/6/2012
Rm	0.200%	3.230%	-13.870%	16.670%	541	
CSAD	6.230%	4.570%	0.430%	27.480%	541	
Dubai						1/13/2004-3/6/2012
R_m	0.120%	4.870%	-21.860%	15.510%	414	
CSAD	4.900%	2.230%	1.690%	13.680%	414	
Kuwait						12/5/1995-3/6/2012
R_m	0.180%	2.240%	-13.100%	7.960%	819	
CSAD	6.410%	3.740%	1.690%	29.670%	819	
Qatar						1/14/2003-3/6/2012
R_m	0.280%	4.330%	-24.050%	14.370%	459	
CSAD	3.880%	1.860%	0.740%	16.570%	459	
Saudi Arabia						1/17/1995-3/6/2012
R_m	0.200%	3.480%	-23.220%	13.710%	855	
CSAD	3.470%	2.140%	1.070%	22.800%	855	
Global Variables						1/17/1995-3/6/2012
Dollar Index Return	0.010%	0.690%	-3.950%	3.070%	894	
S&P 500 Return	0.120%	2.510%	-15.770%	12.370%	894	
Crude Oil return (WTI)	0.190%	5.470%	-37.010%	25.180%	894	
FSI	0.037	1.021	-1.256	5.429	894	
T-Bill Rate (TB3)	3.032	2.079	0.010	6.230	894	
VIX	21.735	8.408	9.900	67.640	894	

Note: Table reports the descriptive statistics for daily market index returns and cross sectional return dispersions across all listed stocks in each exchange, respectively. CSAD is the cross-sectional absolute deviation of returns as a measure of return dispersion. *n* is the number of observations. *FSI* is the St. Louis Federal Reserve's Financial Stress Index, *TB3* is the U.S. three-month Treasury bill rate, VIX is the CBOE Volatility Index, and *WTI* is West Texas Intermediate price. The global variables are used according to their stationarity or lack thereof. Bahrain is not included because its data starts in 2009.

Table 2: Estimates of the Static Herding Model with Global Factors

	Abu Dhabi	Dubai	Kuwait	Qatar	Saudi Arabia
α_0	0.0408***	0.0361***	0.0490***	0.0259***	0.0206***
	(0.0029)	(0.0019)	(0.0021)	(0.0011)	(0.0010)
α_1	0.8810***	0.3791***	0.9654***	0.5094***	0.7247***
	(0.2093)	(0.0762)	(0.1887)	(0.0476)	(0.0733)
α_2	-1.9482	-0.1726	-3.6934	-0.9946***	-1.9444***
	(2.3564)	(0.5955)	(3.1362)	(0.3144)	(0.7336)
α_3	2.3389**	0.0559	2.0485	-0.8407**	-0.7916**
	(1.1658)	(0.8897)	(1.6578)	(0.3812)	(0.3292)
α_4	1.0021***	0.1007	-0.0287	0.0464	0.0528
	(0.2306)	(0.1526)	(0.2084)	(0.1404)	(0.1248)
n	541	414	819	459	855
RSS	0.8821	0.1412	1.0201	0.0961	0.2448
log *L*	968.6662	1065.1346	1576.6933	1292.9927	2274.603

Note: The table reports the estimates for $CSAD_t = \alpha_0 + \alpha_1 |R_{m,t}| + \alpha_2 R^2_{m,t} + \alpha_3 R^2_{SP,t} + \alpha_4 R^2_{o,t} + \varepsilon_t$. All estimations are done using the Newey-West heteroskedasticity and autocorrelation consistent (HAC) standard errors. ***, ** and * represent significance at the 1%, 5%, and 10% levels, respectively. The numbers in parentheses are the HAC standard errors. A significant and negative α_2 estimate implies herding. A significant and negative α_3 estimate implies herding with global stock market (S&P 500) while a significant and negative α_4 estimate implies herding with world oil market. SS is the sum of the squared residuals and log *L* is the log likelihood.

Table 3: Estimates for the Herding Models with Global Factors under Regime Switching

	Abu Dhabi	Dubai	Kuwait	Qatar	Saudi Arabia
$\alpha_{0,0}$	0.0342*** (0.0040)	0.0323*** (0.0075)	0.0334*** (0.0014)	0.0248*** (0.0082)	0.0193*** (0.0060)
$\alpha_{0,1}$	0.0249*** (0.0071)	0.0567*** (0.0019)	0.0643*** (0.0070)	0.0500*** (0.0047)	0.1166*** (0.0063)
$\alpha_{0,2}$	0.0917*** (0.0091)	0.4025*** (0.0056)	0.0651*** (0.0000)	0.2357*** (0.0019)	0.4412*** (0.0048)
$\alpha_{1,0}$	0.2315*** (0.0065)	0.4094*** (0.0090)	0.4772*** (0.0062)	0.4737*** (0.0050)	0.4561*** (0.0058)
$\alpha_{1,1}$	2.1790*** (0.0024)	0.2501*** (0.0043)	0.7896*** (0.0053)	0.7177*** (0.0084)	5.2284*** (0.0028)
$\alpha_{1,2}$	0.6016*** (0.0064)	0.3150*** (0.0050)	0.3416*** (0.0037)	-0.5732*** (0.0032)	0.7339*** (0.0049)
Herding coefficients					
$\alpha_{2,0}$	5.6817*** (1.0086)	0.4659*** (0.0073)	0.8387*** (0.0055)	0.4742*** (0.0014)	0.3721*** (0.0004)
$\alpha_{2,1}$	-5.0019*** (1.0062)	1.4831*** (0.0048)	-2.0942*** (0.0094)	1.0438*** (0.0058)	0.3772*** (0.0043)
$\alpha_{2,2}$	-7.3575*** (1.0034)	-0.1782*** (0.0068)	-1.3780*** (0.0079)	-0.5602*** (0.0070)	-2.0457*** (0.0063)
$\alpha_{3,0}$	-1.7054*** (0.0040)	-0.4618*** (0.0051)	3.7011*** (0.0081)	-0.8383*** (0.0018)	-4.9444*** (0.0069)
$\alpha_{3,1}$	-1.9544*** (0.0074)	1.3472*** (0.0039)	-0.3245*** (0.0037)	-6.1979*** (0.0054)	-1.2506*** (0.0039)
$\alpha_{3,2}$	-1.9432*** (0.0006)	-0.1690*** (0.0035)	-5.2631*** (0.0040)	-0.9940*** (0.0026)	-2.2495*** (0.0009)
$\alpha_{4,0}$	2.3545*** (0.0043)	0.6840*** (0.0041)	-2.0766*** (0.0020)	-0.6002*** (0.0073)	-0.0629*** (0.0050)
$\alpha_{4,1}$	2.3359*** (0.0032)	-9.6987*** (0.0095)	-0.7575*** (0.0001)	-14.0962*** (0.0088)	-0.4912*** (0.0026)
$\alpha_{4,2}$	-2.3385*** (0.0010)	0.0569*** (0.0054)	-0.3687*** (0.0058)	-0.8361*** (0.0080)	-4.2943*** (0.0098)
Regime volatilities					
σ_0	0.0106*** (0.0001)	0.0149*** (0.0001)	0.0083*** (0.0001)	0.0113*** (0.0000)	0.0070*** (0.0000)
σ_1	0.0147*** (0.0000)	0.0244*** (0.0001)	0.0215*** (0.0001)	0.0248*** (0.0000)	0.0167*** (0.0000)
σ_2	0.0438*** (0.0000)	0.0498*** (0.0000)	0.0302*** (0.0000)	0.1269*** (0.0000)	0.0259*** (0.0001)
Time-varying transition probabilities					
$\theta_{00,0}$	1.9649*** (0.0001)	-1.8707*** (0.0001)	2.3275*** (0.0001)	-2.1036*** (0.0000)	0.6833*** (0.0000)
$\theta_{00,1}$	5.1575*** (0.0013)	2.0025*** (0.0056)	-0.8205*** (0.0043)	-48.1803*** (0.0081)	-1.7001*** (0.0013)
$\theta_{00,2}$	-1.9555*** (0.0006)	1.9822*** (0.0039)	-7.7969*** (0.0007)	-2.0059*** (0.0085)	-2.0022*** (0.0077)
$\theta_{00,3}$	24.0772*** (0.0057)	-2.0009*** (0.0046)	-2.0035*** (0.0038)	-2.0001*** (0.0092)	-21.5453*** (0.0071)
$\theta_{00,4}$	0.0560*** (0.0082)	0.6754*** (0.0087)	0.0801*** (0.0054)	0.1136*** (0.0018)	0.0561*** (0.0048)
$\theta_{00,5}$	0.1988*** (0.0055)	-5.7360*** (0.0006)	0.1699*** (0.0094)	0.4050*** (0.0000)	-3.0321*** (0.0071)
$\theta_{00,6}$	-0.0169*** (0.0021)	-1.8101*** (0.0071)	-2.6394*** (0.0021)	0.4999*** (0.0035)	-0.0183* (0.0097)
$\theta_{10,0}$	-1.1022*** (0.0055)	0.0040 (0.0038)	0.0052 (0.0049)	-0.0165** (0.0069)	-4.1387*** (0.0014)
$\theta_{10,1}$	-2.9121*** (0.0020)	0.0113*** (0.0003)	-1.3397*** (0.0097)	9.6787*** (0.0097)	-4.4930*** (0.0090)
$\theta_{10,2}$	0.4066*** (0.0019)	-4.8876*** (0.0027)	1.4041*** (0.0041)	-0.0185*** (0.0052)	-0.0112 (0.0081)
$\theta_{10,3}$	5.0621*** (0.0097)	-0.0020 (0.0066)	0.1196*** (0.0032)	-0.0003 (0.0042)	13.5002*** (0.0083)
$\theta_{10,4}$	1.9746*** (0.0050)	-0.0073 (0.0077)	8.0753*** (0.0025)	-2.0357*** (0.0047)	0.4464*** (0.0050)
$\theta_{10,5}$	0.1930*** (0.0041)	-19.0092*** (0.0007)	4.5113*** (0.0013)	0.0151** (0.0064)	0.0331*** (0.0026)
$\theta_{10,6}$	1.1365*** (0.0013)	-0.0022 (0.0021)	-0.3685*** (0.0040)	0.0007 (0.0033)	1.1764*** (0.0041)
$\theta_{01,0}$	0.7239*** (0.0013)	-0.0046 (0.0033)	-5.2878*** (0.0015)	-1.5341*** (0.0099)	0.0041 (0.0094)
$\theta_{01,1}$	-0.1914*** (0.0041)	-0.0763*** (0.0074)	-0.1180*** (0.0086)	4.2958*** (0.0055)	-0.2451*** (0.0008)
$\theta_{01,2}$	-1.7228*** (0.0049)	74.4895*** (0.0020)	-4.8222*** (0.0012)	-0.8113*** (0.0096)	1.2487*** (0.0049)
$\theta_{01,3}$	-0.3585*** (0.0081)	0.0744*** (0.0069)	1.1853*** (0.0098)	0.0059 (0.0054)	2.0612*** (0.0005)
$\theta_{01,4}$	-0.0268*** (0.0075)	0.4831*** (0.0045)	0.6549*** (0.0060)	0.8529*** (0.0021)	0.2062*** (0.0010)
$\theta_{01,5}$	0.9478*** (0.0058)	49.7426*** (0.0087)	-5.4504*** (0.0012)	0.7373*** (0.0052)	-2.2097*** (0.0097)
$\theta_{01,6}$	0.0436*** (0.0063)	-0.0832*** (0.0053)	-3.8223*** (0.0008)	0.0398*** (0.0037)	0.0454*** (0.0063)
$\theta_{11,0}$	-5.8319*** (0.0052)	-1.9550*** (0.0056)	-2.0202*** (0.0019)	1.7997*** (0.0094)	-1.6146*** (0.0037)
$\theta_{11,1}$	-2.0048*** (0.0014)	-2.0000*** (0.0019)	-2.0140*** (0.0019)	-2.0168*** (0.0049)	-1.9990*** (0.0041)
$\theta_{11,2}$	-2.0002*** (0.0030)	1.9727*** (0.0099)	-11.2699*** (0.0067)	-2.0014*** (0.0045)	2.0023*** (0.0045)
$\theta_{11,3}$	25.3151*** (0.0064)	-2.0001*** (0.0029)	2.0001*** (0.0058)	2.0000*** (0.0095)	-1.9926*** (0.0066)
$\theta_{11,4}$	0.3907*** (0.0010)	-0.6984*** (0.0010)	0.3638*** (0.0011)	-0.4868*** (0.0030)	-0.1604*** (0.0052)
$\theta_{11,5}$	-1.3061*** (0.0086)	-0.7395*** (0.0067)	-0.2036*** (0.0018)	0.2951*** (0.0005)	11.8246*** (0.0079)
$\theta_{11,6}$	0.0256*** (0.0095)	-0.2142*** (0.0095)	-12.3370*** (0.0047)	0.2220*** (0.0020)	-0.1426*** (0.0070)

Table 3: (Continued)

	Abu Dhabi	Dubai	Kuwait	Qatar	Saudi Arabia
$\theta_{02,0}$	1.1263*** (0.0086)	4.6602*** (0.0071)	-0.0047 (0.0070)	0.0007 (0.0030)	4.5218*** (0.0071)
$\theta_{02,1}$	0.0051 (0.0051)	-0.0001*** (0.0000)	-0.1575*** (0.0078)	-0.0271*** (0.0040)	0.0015 (0.0049)
$\theta_{02,2}$	0.0004 (0.0079)	0.0214*** (0.0015)	22.7799*** (0.0022)	0.0015 (0.0024)	-0.0035 (0.0024)
$\theta_{02,3}$	47.4892*** (0.0089)	-0.0005 (0.0033)	-0.0008 (0.0065)	-0.0002 (0.0003)	-0.0946*** (0.0082)
$\theta_{02,4}$	0.0072 (0.0075)	-0.0002 (0.0011)	-0.0606*** (0.0024)	0.0709*** (0.0047)	-0.0673*** (0.0033)
$\theta_{02,5}$	0.0029 (0.0048)	0.1522*** (0.0091)	-6.1147*** (0.0080)	0.0048 (0.0052)	0.0254*** (0.0017)
$\theta_{02,6}$	-8.1586*** (0.0030)	-0.0002 (0.0002)	0.0109* (0.0059)	-0.0003 (0.0018)	-0.5829*** (0.0088)
$\theta_{12,0}$	6.5710*** (0.0012)	-0.0031 (0.0056)	0.0027 (0.0023)	2.5116*** (0.0044)	-0.0002 (0.0070)
$\theta_{12,1}$	-0.1600*** (0.0054)	-0.0190*** (0.0045)	-4.3725*** (0.0086)	1.1598*** (0.0041)	-0.7983*** (0.0014)
$\theta_{12,2}$	-0.1404*** (0.0059)	-5.7212*** (0.0044)	0.1915*** (0.0047)	0.4691*** (0.0076)	0.4702*** (0.0037)
$\theta_{12,3}$	1.3386*** (0.0081)	0.0170*** (0.0015)	-0.0647*** (0.0056)	0.0046 (0.0084)	0.1401*** (0.0016)
$\theta_{12,4}$	0.7060*** (0.0072)	-0.0210** (0.0084)	-3.6273*** (0.0092)	-0.2169*** (0.0079)	-2.6981*** (0.0037)
$\theta_{12,5}$	-0.1035*** (0.0023)	18.8660*** (0.0075)	1.5702*** (0.0096)	-1.8156*** (0.0012)	0.1518*** (0.0089)
$\theta_{12,6}$	0.7073*** (0.0045)	-0.0066* (0.0036)	-2.7105*** (0.0083)	-0.0016 (0.0030)	-2.1844*** (0.0077)
τ_0	6.48	19.64	33.89	3.63	13.48
τ_1	5.93	11.36	6.72	2.20	2.43
τ_2	2.27	7.98	7.86	3.86	2.60
n	541	414	819	459	855
n_0	204.56	143.63	294.99	165.35	371.62
n_0	201.11	138.01	258.72	124.56	236.49
n_0	135.33	132.37	265.30	169.09	246.89
LR	786.487*** (0.000) [0.000]	287.080*** (0.000) [0.000]	1122.538*** (0.000) [0.000]	305.509*** (0.000) [0.000]	973.822*** (0.000) [0.000]
AIC	-2687.8192	-2381.3491	-4239.9246	-2855.4943	-5474.9362
log L	1361.9096	1208.6745	2137.9623	1445.7472	2761.5138

Notes: This table presents the estimates of the 3-regime TVTP-MSH model given in Equations (3) through (5). Robust standard errors are reported in parentheses, which are obtained using the sandwich estimator of Huber (1967) and White (1982) based on the outer product of gradients and the second derivative matrix. n is the total number of observations, n_k is the number of observations in regime k, τ_k is the duration of regime k, and LR test is the linearity test. The LR test is nonstandard since there are unidentified parameters under the null. The χ^2 p-values with degrees of freedom equal to the number of restrictions plus the number of parameters unidentified are given in parentheses and the p-values of Davies (1987) test are given in square brackets. The asterisks ***, ** and * represent significance at the 1%, 5%, and 10% levels, respectively.

Appendix

When there are more than two states in the MS herding model as in Equations (4) and (5), it is not straightforward to restrict the column sum of transition probabilities to be equal to one and each probability to lie between 0 and 1. For a 2-state MS model the restrictions are easier to impose. Following Perez-Quiros and Timmermann (2000) one can in this case use normal cumulative density (CDF) function transformation to get probabilities between 0 and 1. However in our case, a recursive TVTP generating function is used to constrain for each probability to be estimated for the 3-state MS models.

The recursive probability generating is based on first estimating the probability components, $\tilde{p}_{ij,t}$ and then transforming the components into TVTPs. For a *k*-state MS model, there are (*k*-1)*k* independent TVTPs each and for each one we define the probability component as

$$\tilde{p}_{ij,t} = \Phi(\mathbf{Z}_{ij,t-1}\theta_{ij}), \qquad i = 0,1,\ldots,k-2,\ j = 0,1,\ldots,k-1 \tag{A1}$$

where t=1,2, …, T is the time, $\Phi(\cdot)$ is the normal cumulative distribution function (CDF) and $\mathbf{Z}_{ij,t}$ is the vector of state-variables that impacts the transition from state *j* to state *i*, and θ_{ij} are the parameters that need to be estimated jointly with the parameters in Equation (4). The probability components are collected into the following matrix:

$$\tilde{P}_t = \begin{bmatrix} \tilde{p}_{01,t} & \tilde{p}_{01,t} & \cdots & \tilde{p}_{0,k-1,t} \\ \tilde{p}_{10,t} & \tilde{p}_{11,t} & \cdots & \tilde{p}_{1,k-1,t} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{p}_{k-2,0,t} & \tilde{p}_{k-2,1,t} & \cdots & \tilde{p}_{k-2,k-1,t} \\ 1 & 1 & \cdots & 1 \end{bmatrix} \tag{A2}$$

Using the matrix $\tilde{P}_t$, which contains the probability components $\tilde{p}_{ij,t}$ we can obtain the matrix P_t which contains the final TVTPs denoted $p_{ij,t}$, from the following transformation:

$$P_t = \tilde{P}_t \circ R_t = \begin{bmatrix} p_{00,t} & p_{01,t} & \cdots & p_{0,k-1,t} \\ p_{10,t} & p_{11,t} & \cdots & p_{1,k-1,t} \\ \vdots & \vdots & \ddots & \vdots \\ p_{k-1,0,t} & p_{k-1,1,t} & \cdots & p_{k-1,k-1,t} \end{bmatrix} \tag{A3}$$

where $\circ$ is entrywise or Hadamard matrix product and R_t is an auxiliary matrix defined as

$$R_t = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1-\tilde{p}_{00,t} & \tilde{p}_{01,t} & \cdots & \tilde{p}_{0,k-1,t} \\ \vdots & \vdots & \ddots & \vdots \\ \prod_{i=1}^{k-3}(1-\tilde{p}_{i0,t}) & \prod_{i=1}^{k-3}(1-\tilde{p}_{i1,t}) & \cdots & \prod_{i=1}^{k-3}(1-\tilde{p}_{i,k-1,t}) \\ \prod_{i=1}^{k-2}(1-\tilde{p}_{i0,t}) & \prod_{i=1}^{k-2}(1-\tilde{p}_{i1,t}) & \cdots & \prod_{i=1}^{k-2}(1-\tilde{p}_{i,k-1,t}) \end{bmatrix} \tag{A4}$$

It should be noted that $\tilde{p}_{ij,t}$ is not the final TVTP estimate, it is rather a component of it. The transformation in Equation (A4) yields the following estimate of $p_{ij,t}$:

$$\begin{aligned} p_{0j,t} &= \tilde{p}_{0j,t} \\ p_{1j,t} &= (1-\tilde{p}_{0j,t})\tilde{p}_{1j,t} \\ &\vdots \\ p_{k-2,j,t} &= (1-\tilde{p}_{0j,t})(1-\tilde{p}_{1j,t})\cdots(1-\tilde{p}_{k-3,j,t})\tilde{p}_{k-1,j,t} \\ p_{k-1,j,t} &= (1-\tilde{p}_{0j,t})(1-\tilde{p}_{1j,t})\cdots(1-\tilde{p}_{k-3,j,t})(1-\tilde{p}_{k-2,j,t}) \end{aligned} \tag{A5}$$

for column $j = 0,1,\ldots,k-1$. The transformation as defined in (A5) automatically restricts each $p_{ij,t}$ to lie between 0 and 1 and the columns of matrix P_t sum to 1. Therefore, one does not need to use constrained optimization. The maximum likelihood estimates of the TVTP-MS model in Equations (4) and (5) can be obtained using unconstrained optimization using the transformation in Equations (A3) and (A5).